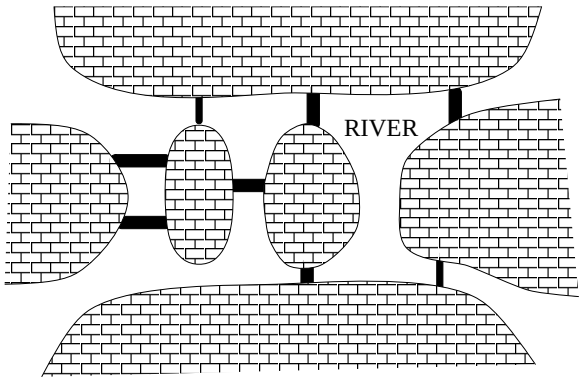
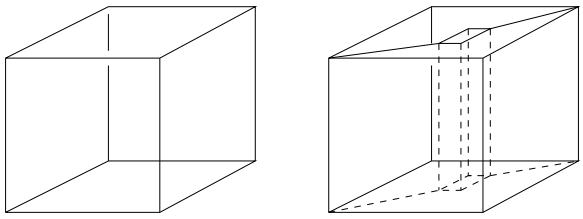
History of topology problems

1. Does the following town admit a tour starting and ending at the same point and crossing each bridge precisely once? Could such a tour be made if precisely *one* extra bridge were built in an appropriate position? Could such a tour be made if precisely *two* extra bridegs were built? Explain your answers carefully and state precisely any theorem of Euler that you use.



2. Determine the Euler characteristic $\chi = F - E + V$ for (a) the surface of a solid cube and (b) the surface of a solid cube with a rectangular hole cut between opposite faces.



Are either of these polyhedra convex?

3. A double torus (i.e. the surface of a doughnut with two holes) is divided into polygonal regions using two vertices and eight edges. How many regions are there?

4. Using a DNA array with probes of length 3, an unknown DNA sequence was found to have precisely the following fragnments

$$S = \{TGG, ATG, CGT, GCG, GCA, GGC, GTG, TGC\}$$

of length 3. Construct all possibilities for the unknown DNA sequence by finding Euler paths in a suitable graph.