Example For the real line $\mathbb{E}^1$ we have the subset $\mathbb{Q} \subseteq \mathbb{E}^1$ of rational numbers. With the subspace topology on $\mathbb{Q}$ we see that $\mathbb{Q}$ is not connected. For instance

$$A = \{x \in \mathbb{R} : x > \sqrt{2}\} = (\sqrt{2}, \infty)$$

$$B = \{x \in \mathbb{R} : x < \sqrt{2}\} = (-\infty, \sqrt{2})$$

Then

$$\mathbb{Q} = (A \cap \mathbb{Q}) \cup (B \cap \mathbb{Q})$$

and

$$(A \cap \mathbb{Q}) \cap (B \cap \mathbb{Q}) = \emptyset$$

and both $A \cap \mathbb{Q}$ and $B \cap \mathbb{Q}$ are open subsets of $\mathbb{Q}$.

The connected components of $\mathbb{Q}$ are the ~~sets~~ subspaces

$\{x\}$ for each $x \in \mathbb{Q}$.

Introduction to topological data analysis

	H	M	R	C	W
H	0	11	10	14	22
M	11	0	3	13	21
R	10	3	0	12	20
C	14	13	12	0	16
W	22	21	20	16	0

- Human, Mouse, Rat, Cat, Whale
- Halfords, M&S, River Island, Currys, Woodies

$$\mathrm{dist}(H, M) = \mathrm{dist}(M, H)$$

$$\mathrm{dist}(H, H) = 0$$

Choose some number $\varepsilon > 0$, called a threshold, and consider

the graph G_ε with vertices

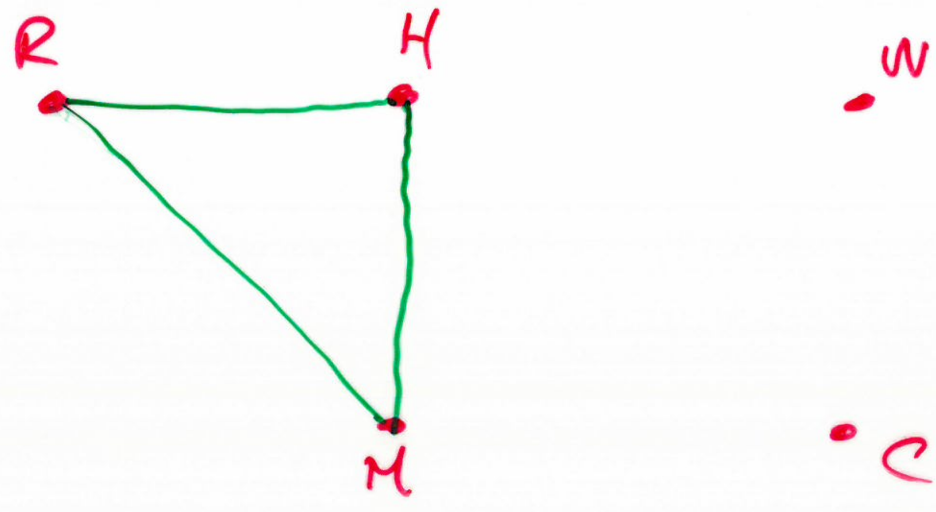
H, M, R, C, W

and with an edge

if $\mathrm{dist}(X, Y) \leq \varepsilon$, $X = X$.

e.g. for $\varepsilon = 11$

G_{11}



we regard this graph as a subspace of $\mathbb{R}^5 = \mathbb{E}^5$ by identifying

$H = (1, 0, 0, 0, 0] = e_1$

$M = (0, 1, 0, 0, 0) = e_2$

$R = (0, 0, 1, 0, 0) = e_3$

$C = (0, 0, 0, 1, 0) = e_4$

$W = (0, 0, 0, 0, 1) = e_5$

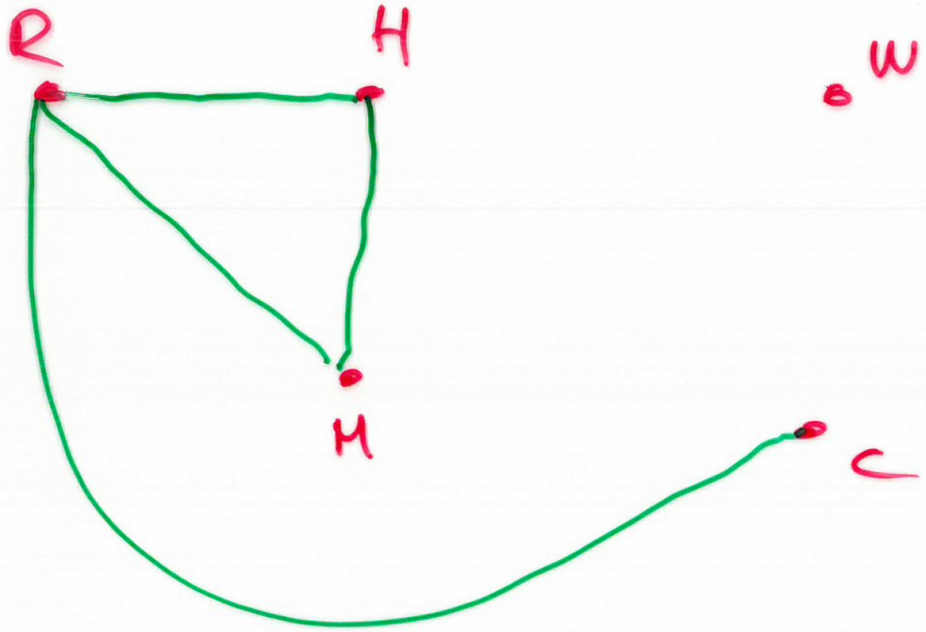
The graph G_{11} can be thought of as a subspace of $\mathbb{E}^5$ with points e_1, e_2, e_3, e_4, e_5 and line segments

$$e_1 e_3, \quad e_2 e_3, \quad e_1 e_2.$$

The graph G_{11} has three connected components

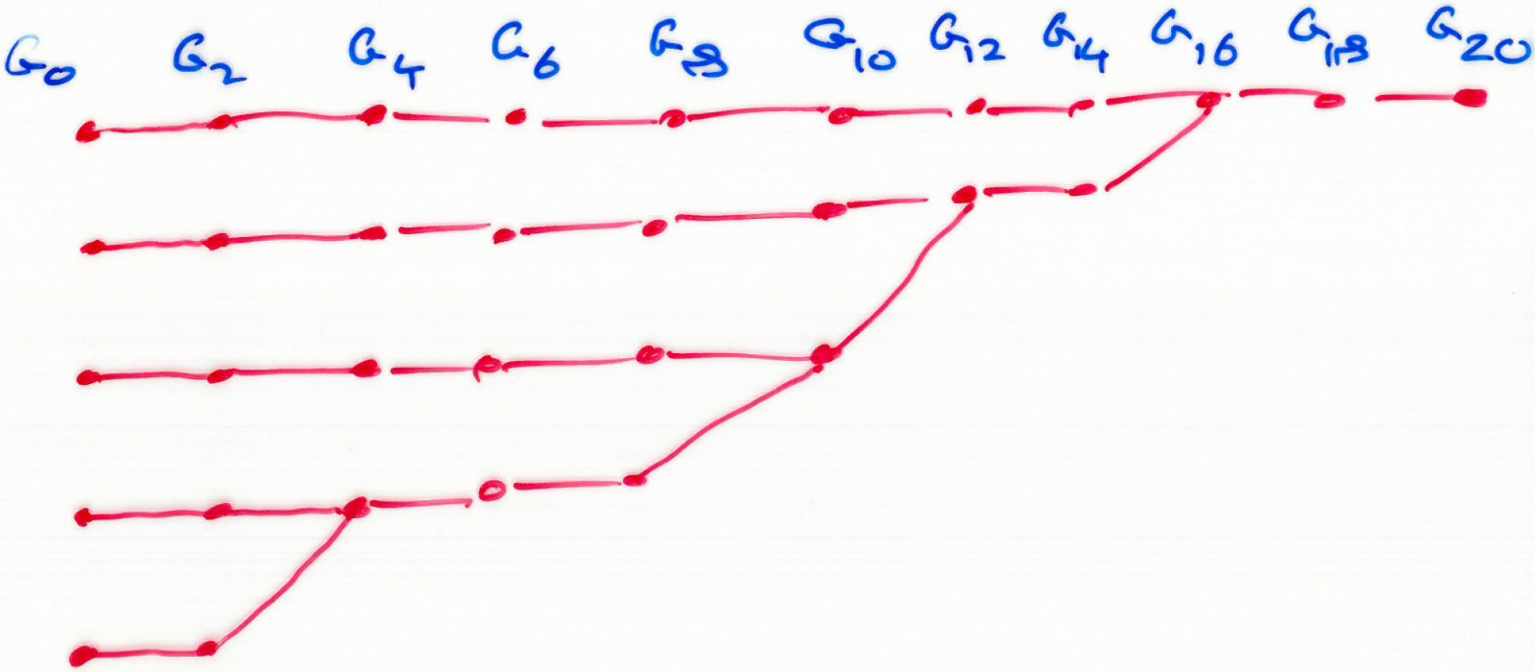
$$X_W, \quad X_C, \quad X_{RHM}$$

Now let's look at G_{12}:



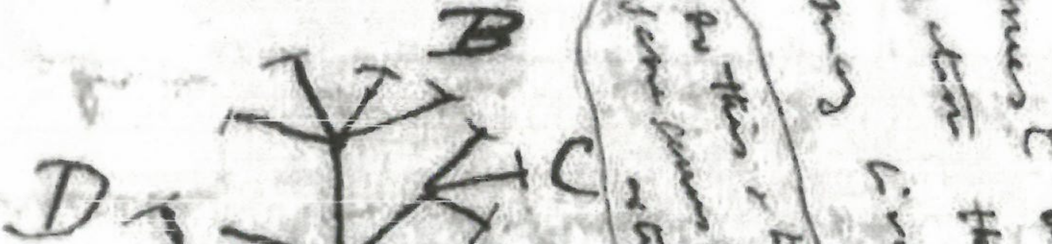
So G_{12} has <u>two</u> connected components, X_W, X_{RHMC}

A <u>dendragram</u> summarizes the inclusions of connected components:



I think

Case must be that one generation then should be as many living as now. To do this & to have many species in same genus (as is) requires extinction.



Continuity

Defn Let X, Y be topological spaces. A function $f : X \to Y$ is *continuous* if the inverse image of every open set in Y is an open set in X.

In other words, if $U \subseteq Y$ is an open subset of Y then

$$f^{-1}(U) = \{x \in X : f(x) \in U\}$$

is open in X.

Example Consider

$X = \{a, b, c\}$, $T = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

$Y = \{a, b, c, d\}$, $T = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$

Is the function

$$f: X \to Y$$

$$f(a) = a$$

$$f(b) = b$$

$$f(c) = c$$

continuous?