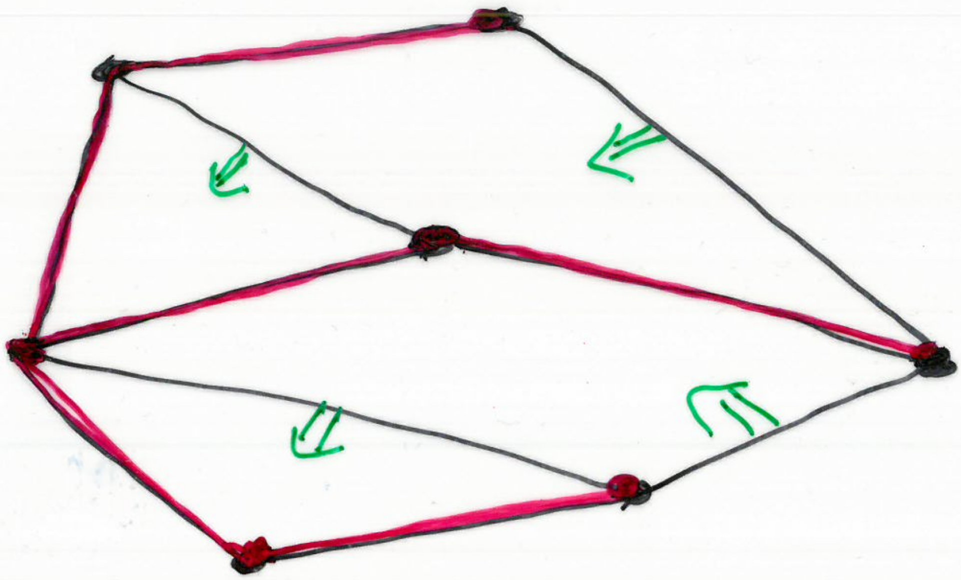
Consider a planar graph G



with V vertices, E edges and F faces.

Let T be some maximal tree in the graph. Let ~~T_V and~~ V_T and E_T denote the number of vertices and edges in T.

$V_T = V$

$E_T = V-1$

Each black edge of G not in T forms a loop involving all but one of its edges in T.

now

$$V - E + F =$$

$$V - \left(E_T + \#\,\text{black edges}\right) + F$$

$$= V - (V-1) - (F-1) + F$$

$$= 2.$$

The sphere

$$S^2 = \{ (x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2=1 \}$$

is a more precise notion than the surface of Mars.

Our explanation of the Euler characteristic formula

$$\chi(S^2) = 2$$

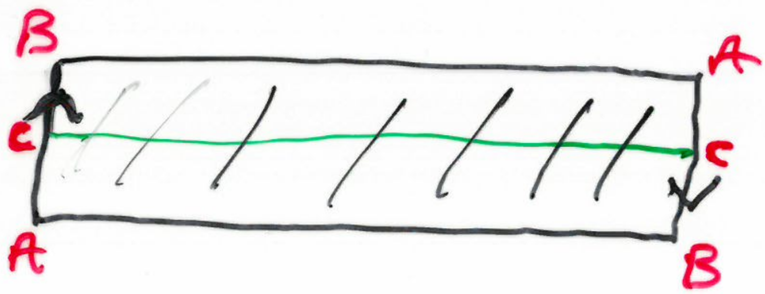
used the fact that any loop on the sphere, with no self intersections

has an inside and an outside.

i.e. any such loop cuts S^2 into two regions

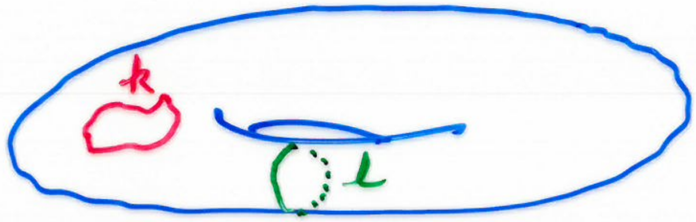
Question: Is this "fact" obvious?

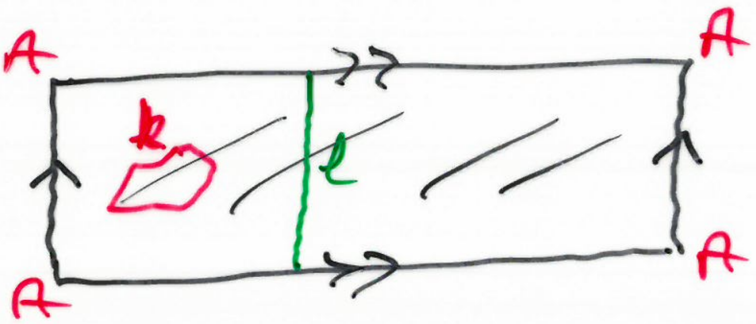
Example Consider the Möbius strip:



Draw a loop around the centre of the strip. With paper, pencil and scissors check that the green loop does not cut the Möbius strip into two pieces.

<u>Example</u> Consider a torus





(or surface of a doughnut).
Loop k cuts the torus into two pieces, but loop l does <u>not</u>.

Example Is it obvious that the following loop in the plane $\mathbb{R}^2$ cuts the plane into two pieces.

Topology offers a precise language and a collection of techniques for studying such questions.

Notation:

$$S^1 = \{ (x,y) \in \mathbb{R}^2 : x^2+y^2=1 \}$$

So S^1 is our notation for a circle.

The MA342 module will enable you to understand the statement of the following result, and also its proof given in Armstrong's text.

Jordan Curve Theorem

Let $\alpha : S^1 \longrightarrow \mathbb{R}^2$ be any injective continuous function. Let $J \subseteq \mathbb{R}^2$ be the image of α. The $\mathbb{R}^2 \setminus J$ has precisely two connected components, both of which have frontier J.

Aim for the next few lectures:

1) Explain above underlined terms

2) Give an outline proof/explanation of the above theorem

3) Give some weird examples that suggest the theorem is not so obvious.