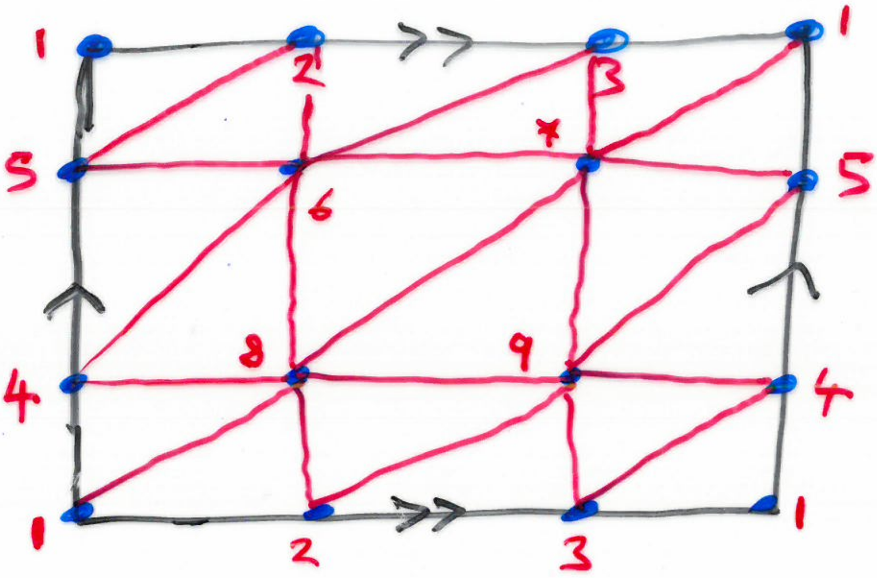
Example Triangulation of the torus:

Torus

$S^1 \times S^1$

$\simeq$



This describes a simplicial complex homeomorphic to the torus.

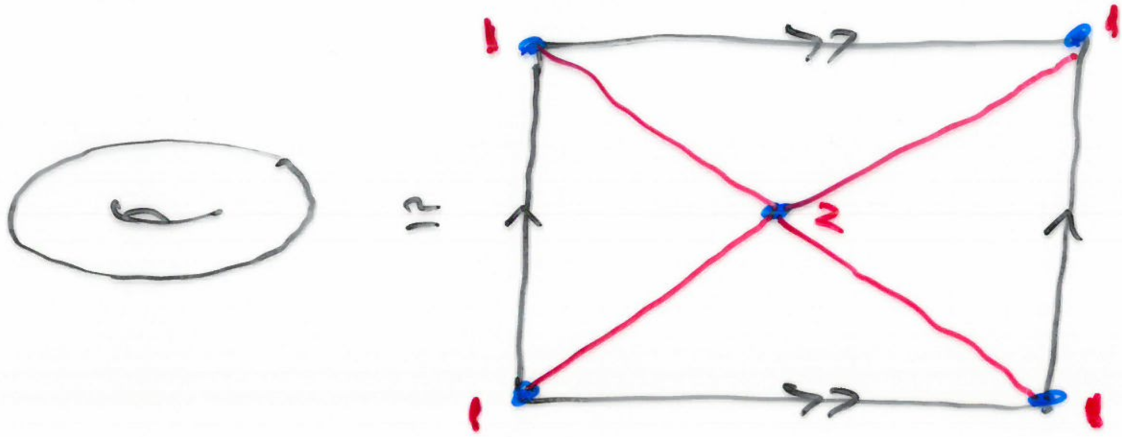
α_k = number of k-simplices

$\alpha_0 = 9$

$\alpha_1 = 27$

$\alpha_2 = 18$

Example (not a triangulation of torus)



There is no 1-simplex with just one vertex!

Definition Let K be a simplicial complex with α_k k-simplices. The Euler characteristic of K is:

$$\chi(K) = \alpha_0 - \alpha_1 + \alpha_2 - \alpha_3 + \cdots$$

Theorem 1 (Most profound in course) If two simplicial complexes K and L are such that $|K|$ is homeomorphic to $|L|$ then

$$\chi(K) = \chi(L).$$

Using this theorem we can make the following

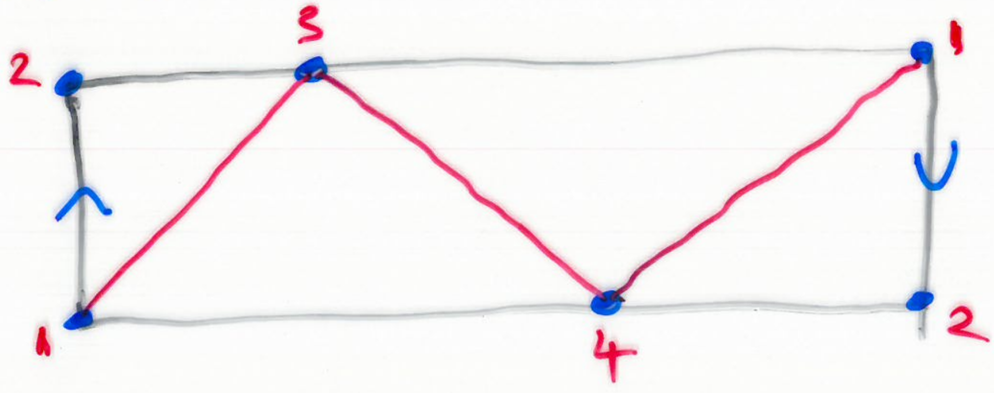
<u>Definition</u>: If X is a topological space with a triangulation K, $h : |K| \xrightarrow{\cong} X$ then we define

$$\chi(X) = \chi(K).$$

<u>Example</u>

$\chi(\text{torus}) = 9 - 27 + 18 = 0$

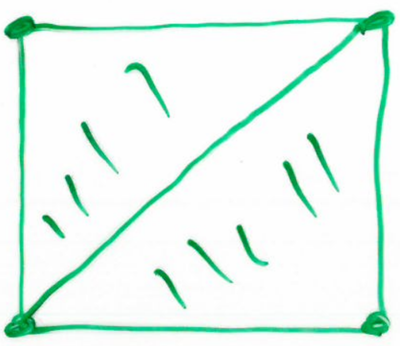
<u>Example</u> Determine the Euler characteristic of the Möbius band.



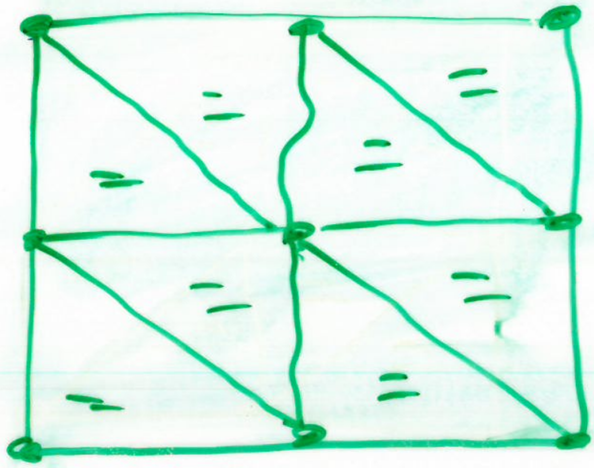
$\chi(\text{Möbius band}) = 4 - 8 + 4 = 0$

Initial attempts at proving Theorem 1 focussed on:

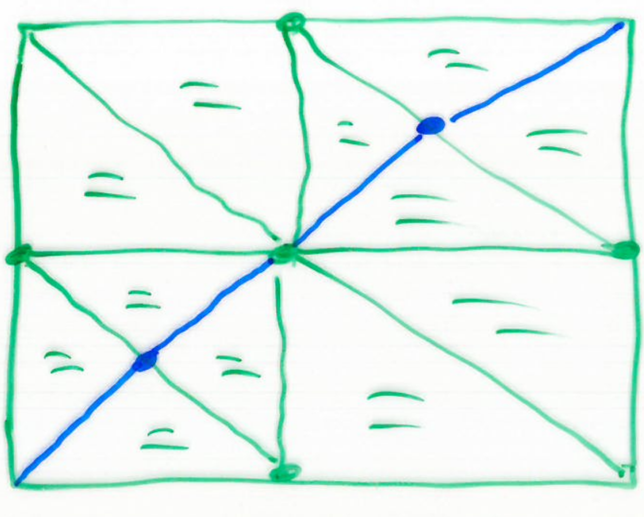
<u>Hauptvermutung</u>: If K and L are triangulations of X then there are subdivisions K' of K and L' of L such that $K' = L'$.



K



L

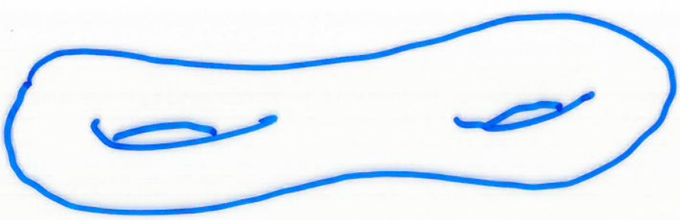


$K' = L'$

The Hauptvermutung was proved for simplicial complexes of dimension ≤ 3 by Morse in the 1950s.

In 1961 John Milnor proved the Hauptvermutung is false in dimensions ≥ 6.

<u>Exercise</u> Determine the Euler characteristic of a "double torus"



by constructing a triangulation of the double torus.