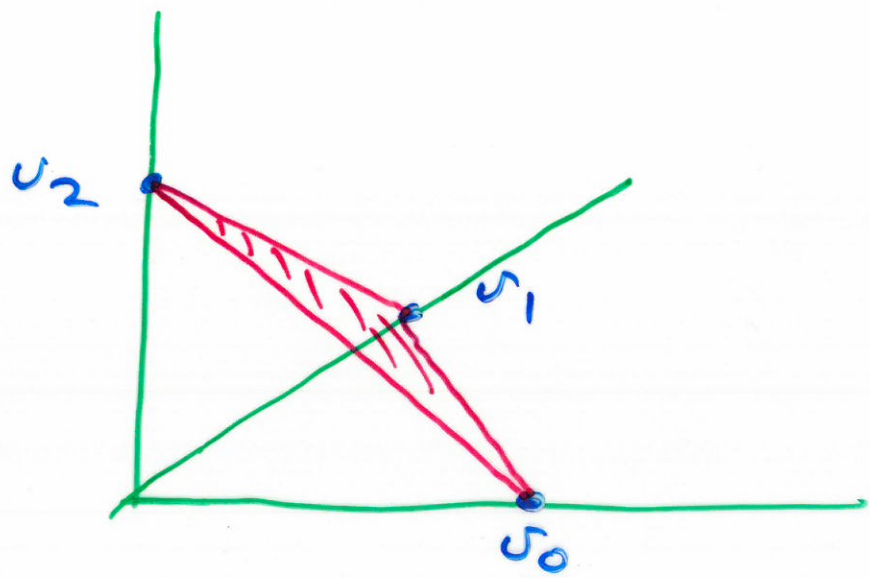
Suppose $v_0, v_1, \dots, v_k \in \mathbb{E}^n$ are in general position. Let

$$C = \text{Convex}(v_0, v_1, \dots, v_k)$$

contain the smallest convex set in $\mathbb{E}^n$ containing $v_0, v_1, \dots, v_k$.

Example $v_0 = (1,0,0)$, $v_1 = (0,1,0)$, $v_2 = (0,0,1)$.



In general, $C = \text{Convex}(v_0, v_1, \dots, v_k)$ consists of all points of the form

$$\mathbb{E}^n \ni x = \lambda_0 v_0 + \lambda_1 v_1 + \dots + \lambda_k v_k$$

with $\lambda_i \in \mathbb{R}$, $\lambda_i \geq 0$ and

$$\lambda_0 + \lambda_1 + \lambda_2 + \dots + \lambda_k = 1.$$

Defn Let $v_0, v_1, \dots, v_k \in \mathbb{E}^n$ be in general position. We call

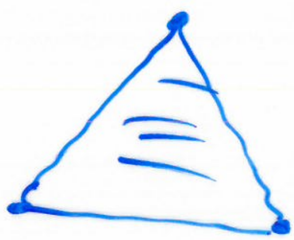
$$C = \mathrm{Convex}(v_0, v_1, \dots, v_k)$$

a simplex of dimension k, or k-simplex.

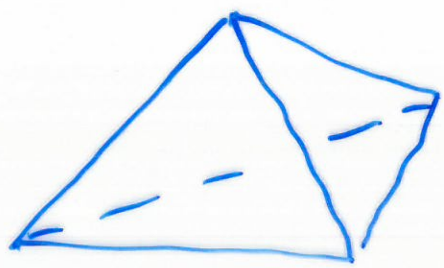
0-simplex = point

1-simplex = line segment

2-simplex = solid triangle



3-simplex = tetrahedron



Simplexes have "faces".

If A and B are simplexes, and if the vertices of A form a subset of the vertices of B, then we say that A is a <u>face</u> of B.

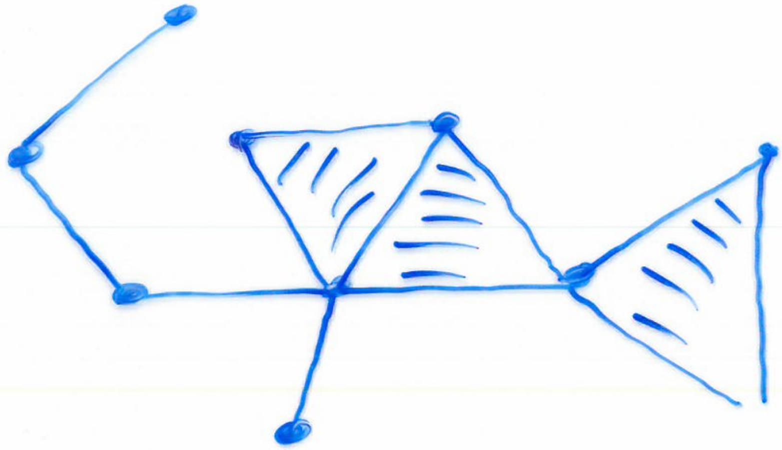
<u>Example</u> A 3-simplex has

- four faces of dimension 2.
- six faces of dimension 1.
- four faces of dimension 0.

Definition A finite collection of simplexes in $\mathbb{E}^n$ is called a simplicial complex if:

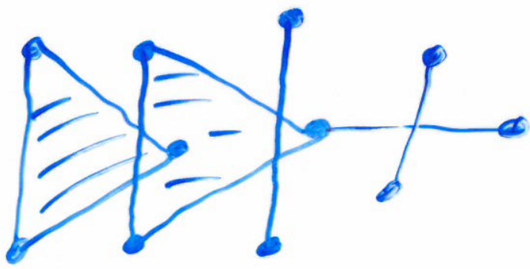
i) whenever a simplex lies in the collection, then so too does all its faces

ii) whenever two simplexes of the collection intersect, they do so in a common face.



simplicial
complex

Not a simplicial
complex



A simplicial complex is a subset of $\mathbb{E}^n$ and is thus a subspace with the subspace topology.

We let $K, L, \ldots$ denote simplicial complexes. We write $|K|, |L|, \ldots$ for the corresponding topological subspaces of Euclidean space.

Defn A triangulation of a topological space X is consists of a simplicial complex K and a homeomorphism

$$h : |K| \longrightarrow X$$