Aim: show that our function

$$f : [0,1] \longrightarrow \Delta$$

is surjective.

It should be clear that

Δ equals the union of image(f) with the accumulation points

of image(f).

So need to show that image(f) contains all its accumulation points.

i.e. Need to show that

image(f) is closed.

We know that $[0,1]$ is compact, and hence that

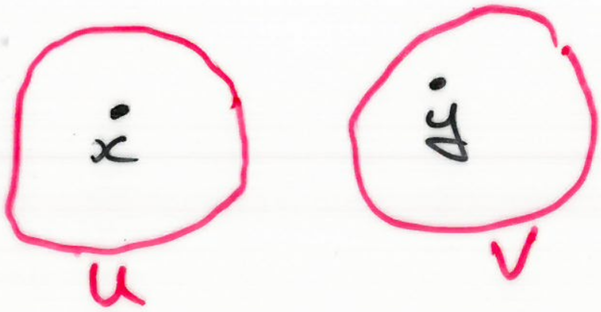
$f([0,1])$ is compact.

$\overset{\shortparallel}{\text{image}}(f)$

we'll show that "compact" implies "closed" under a suitable hypothesis.

<u>Defn</u> A topological space X is said to be <u>Hausdorff</u> if for any distinct $x, y \in X$ there exists open sets $U, V \subseteq X$ with

$x \in U$, $y \in V$, $U \cap V = \emptyset$.



<u>Example</u> $\mathbb{R}$ with the standard topology is Hausdorff. So too is $\mathbb{R}^n$.

__Example__ ~~the~~ Let's give $\mathbb{R}$ the cofinite topology - a set is open iff its complement is finite. This space is __not__ Hausdorff.

__Proposition__ A compact subset of a Hausdorff topological space is closed.

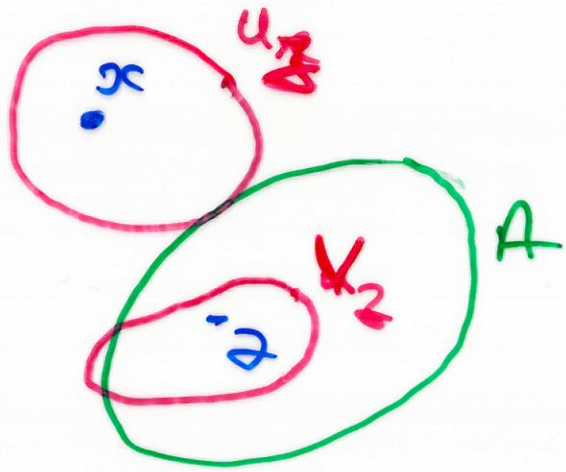
__Proof__ Let X be a Hausdorff topological space. Let A be a compact subset of X.

Let $x \in X \setminus A$. We just need to show that x is __not__ an accumulation point of A.

Let $z \in A$. We can find open sets U_z, V_z such that

$z \in V_z \quad x \in U_z, \quad U_z \cap V_z = \emptyset,$

since X is Hausdorff.



we have a collection of open sets

$$\{ V_z \}_{z \in A}$$

The union of this collection contains A. But A is compact, so (!) we can find a finite collection of points

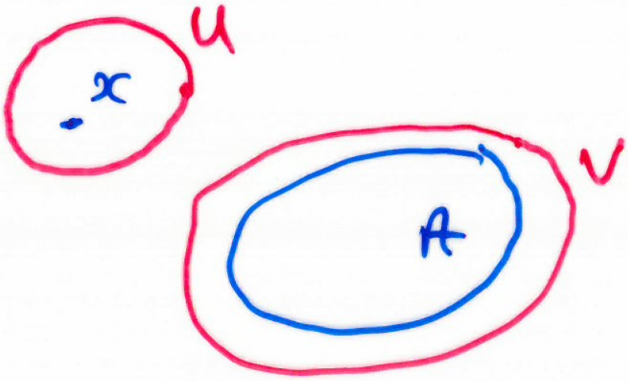
$z_1, z_2, \ldots, z_k \in A$ with

$A \subseteq V_{z_1} \cup V_{z_2} \cup \cdots \cup V_{z_k} = V$ say.

Now V is disjoint from the finite intersection

$$U = U_{z_1} \cap U_{z_2} \cap \cdots \cap U_{z_k}.$$

But U is open since it is a finite intersection of open sets.



Hence $U \cap A = \emptyset$ and so x is <u>not</u> an accumulation point. Hence A contains all its accumulation points and is thus closed. $\square$

Aims:

- Give a precise definition of the Euler characteristic of a space.
- Give a flavour of the ingredients in the proof that Euler charistic is a topological property.
- Give an application of Euler characteristic to economics.

Towards simplicial complexes

Let $v_0, v_1, \ldots, v_k$ be vectors in $\mathbb{E}^n$. These vectors are said to be in <u>general position</u> if the vectors

$$v_1 - v_0,\ v_2 - v_0,\ \ldots,\ v_k - v_0$$

are linearly independent.

<u>Example</u> $v_0 = (1,0,0)$, $v_1 = (0,1,0)$, $v_2 = (0,0,1)$.

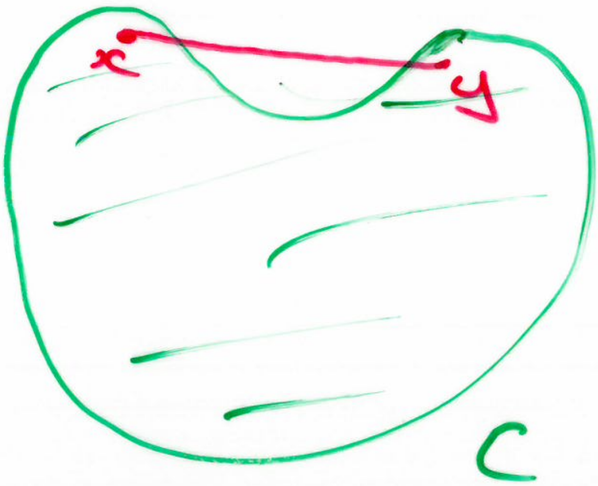
Then

$$v_1 - v_0 = (-1, 1, 0)$$

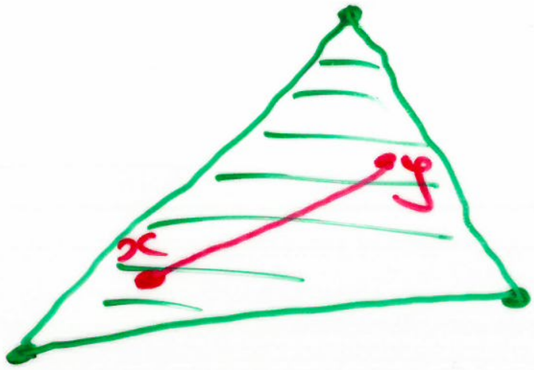
$$v_2 - v_0 = (-1, 0, 1)$$

are linearly independent. Hence v_0, v_1, v_2 <u>are</u> in general position.

<u>Recall:</u> A set $C \subseteq \mathbb{E}^n$ is said to be convex if, for any points $x, y \in C$, all points on the line from x to y lie in C.



not convex



is convex