

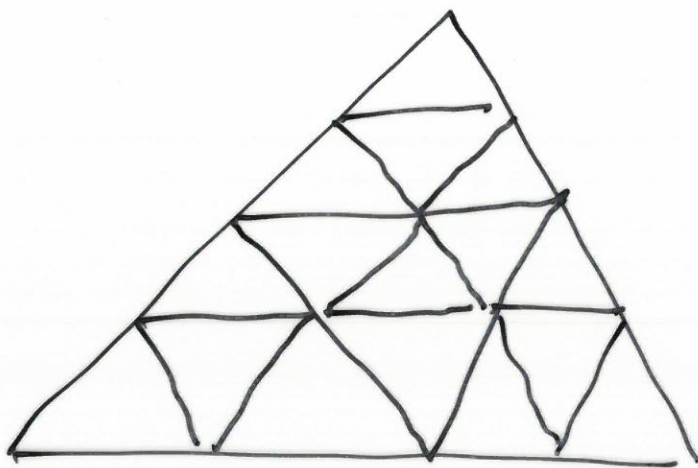
Last time: were constructing

$$f: [0,1] \rightarrow \Delta$$

as

$$f(t) = \lim_{n \rightarrow \infty} f_n(t),$$

where $f_n(t)$ was defined
by subdividing Δ into small
subtriangles



of side $\frac{1}{2^{n-1}}$.

If t is close to t' then
 $f(t)$ is close to $f(t')$, intuitively
 $f(t)$ is continuous. It is

routine to convert this
intuition into a formal
proof of continuity.

It remains to prove
that $f(t)$ is surjective.
For this we'll use
compactness of $[0,1]$
and "related results".

Some Theory

Definition: Let X be a topological space, A subset $A \subseteq X$ is said to be closed if its complement $X \setminus A$ is open.

Example $[0, 1] \subseteq \mathbb{R}$ is closed with respect to the usual topology on $\mathbb{R}$ since

$$\mathbb{R} \setminus [0, 1] = (-\infty, 0) \cup (1, \infty)$$
is open.

Example $(0, 1]$ is neither open nor closed.

Definition Let A be a subset of a topological set X . A point $p \in X$ is an accumulation point of A if every open subset $U \subseteq X$ that contains p also contains some point in $A \setminus \{p\}$.

Example Let $X = \mathbb{R}$ with standard topology. Consider

$$A = \left\{ \frac{1}{n} \right\}_{n=1,2,3,\dots}$$

In this example 0 is the only accumulation point of A .

Example Let $X = \mathbb{R}$ and $A = [0, 1)$. Then every point in A is an accumulation point of A , and so too is 1.

Proposition A set A in a topological space X is closed if, and only if, it contains all its accumulation points.

Proof Suppose A is closed.

Then $X \setminus A$ is open.

Any point $x \in X \setminus A$

lies in the open set $X \setminus A$,
So no point $x \in X \setminus A$ is
an accumulation point
of A , since $X \setminus A \cap A = \emptyset$.
So A must contain all of
its accumulation points.

Conversely, suppose
that A contains all of
its accumulation points.

Let $x \in X \setminus A$. We
can find an open set
 $U_x \subseteq X \setminus A$ such that
 $x \in U_x$ because x is
not an accumulation
point.

So

$$X \setminus A = \bigcup_{x \in X \setminus A} U_x$$

So $X \setminus A$ is open since
each U_x is open.

Hence A is closed.

