<u>Defn</u> Let $X$ be a set with a topology $\mathcal{T}$. Let $Y \subseteq X$ be a subset. In the <u>subspace topology</u> on $Y$ a subset $U \subseteq Y$ is open if

$$U = Y \cap A$$

where $A$ is an open subset of $X$.

<u>Example</u> For the real line $\mathbb{E}^1$ we have the subset $\mathbb{Q} \subset \mathbb{E}^1$ of rational numbers. With the subspace topology on $\mathbb{Q}$ we see that $\mathbb{Q}$ is <u>not</u> connected.

To see this consider

$$A = \{x \in \mathbb{R} : x > \sqrt{2}\} = (\sqrt{2}, \infty)$$

$$B = \{x \in \mathbb{R} : x < \sqrt{2}\} = (-\infty, \sqrt{2})$$

Then

$$\mathbb{Q} = (A \cap \mathbb{Q}) \cup (B \cap \mathbb{Q})$$

and

$$(A \cap \mathbb{Q}) \cap (B \cap \mathbb{Q}) = \phi$$

and

$A \cap \mathbb{Q}$ is open in the subspace topology, as is $B \cap \mathbb{Q}$.

**Definition** A connected component of a topological space $X$ is a connected subspace $Y \subseteq X$ such that there is no connected subspace $W \subseteq X$ with $Y \subsetneq W$.

**Example**

Let $X = \{(x,y) \in \mathbb{E}^2 : x^2+y^2 \neq 1\}$

There are two connected components of $X$, namely

$$Y = \{(x,y) \in \mathbb{E}^2 : x^2+y^2 < 1\}$$

and

$$Z = \{(x,y) \in \mathbb{E}^2 : x^2+y^2 > 1\}.$$

$Y =$

$Z =$

<u>Example</u>

Let $X = \mathbb{Q}$ and let $\mathbb{Q} \subset \mathbb{E}^1$ be endowed with the subspace topology. Then the connected components of $\mathbb{Q}$ are the sets $\{x\}$ for each $x \in \mathbb{Q}$. There are (countably) infinitely many connected components of $\mathbb{Q}$.