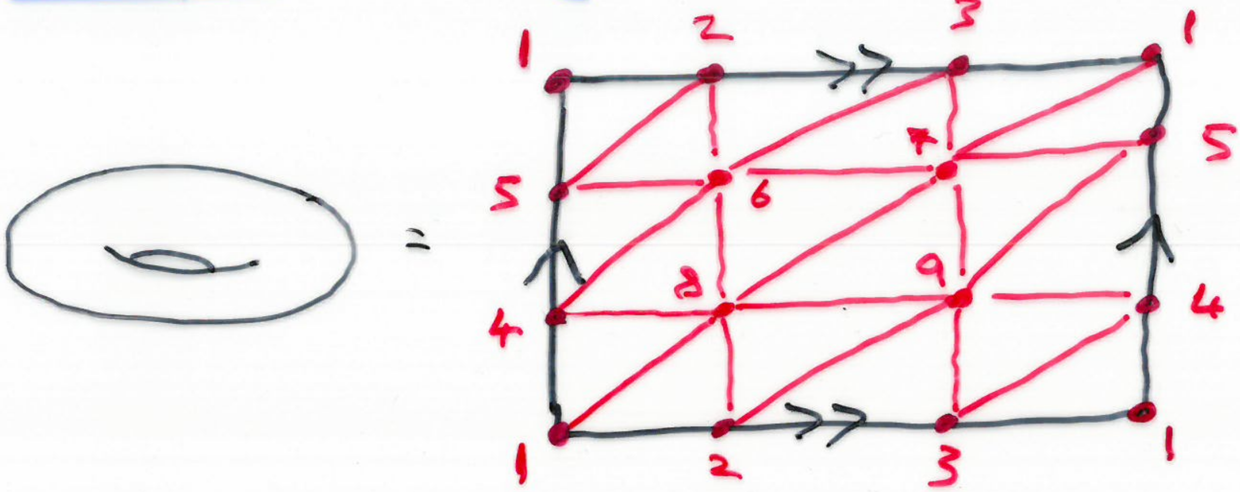
Example Triangulation of the torus:



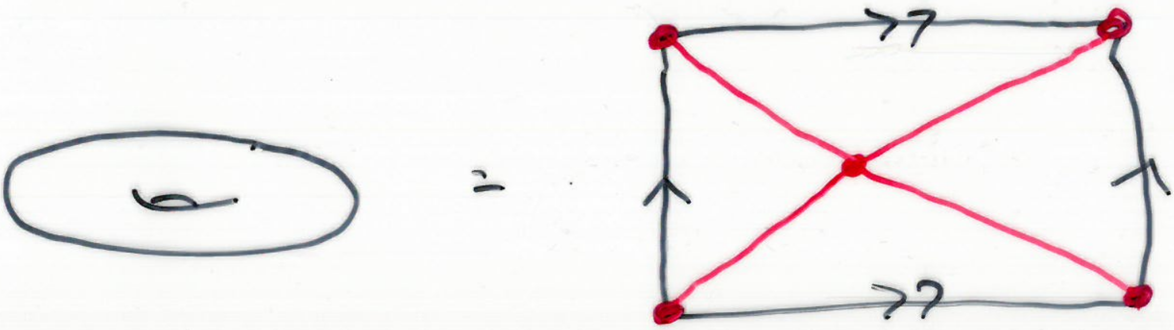
α_k = number of k-simplices

$\alpha_0 = 9$

$\alpha_1 = 27$

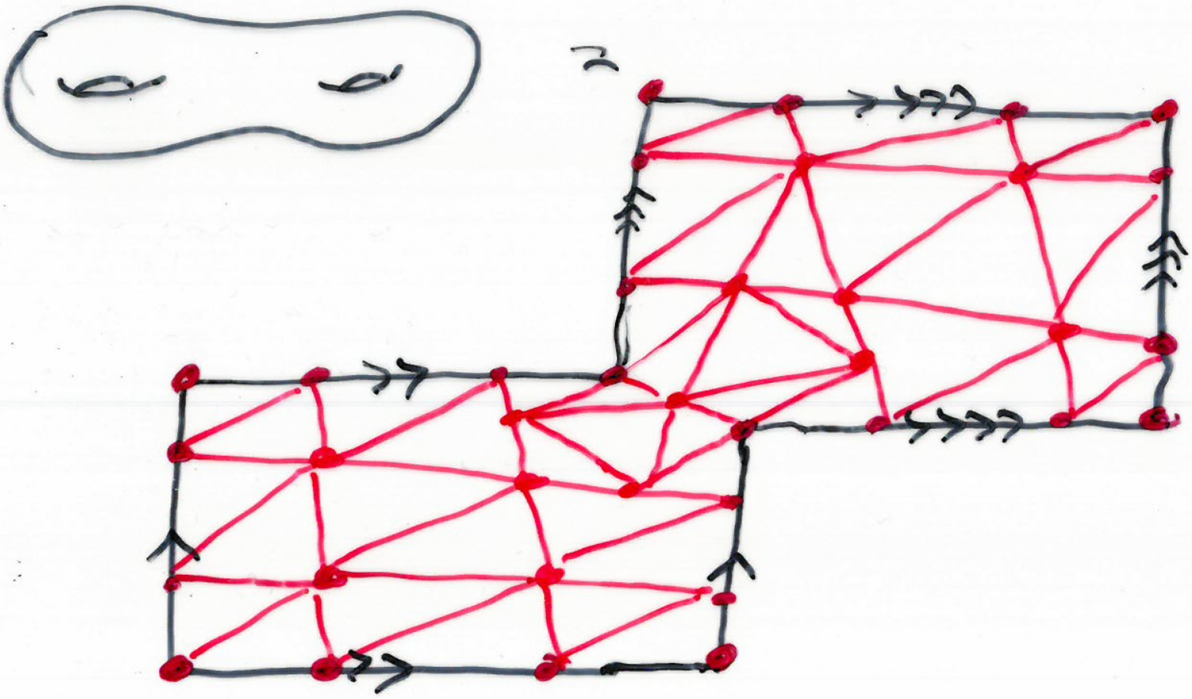
$\alpha_2 = 18$

Example (non-example)



This is not a triangulation of the torus because a k-simplex must have $k+1$ vertices.

<u>Example</u> Triangulation of a "double torus"



$\alpha_0 =$

$\alpha_1 =$ exercise

$\alpha_2 =$

<u>Definition</u> Let K be a simplicial complex with α_k k-simplices. The <u>Euler</u> <u>characteristic</u> of K is

$$\chi(K) = \alpha_0 - \alpha_1 + \alpha_2 - \alpha_3 + \alpha_4 - \cdots \quad .$$

Theorem If two simplicial complexes K and L are such that $|K|$ is homeomorphic to $|L|$ then

$$\chi(K) = \chi(L).$$

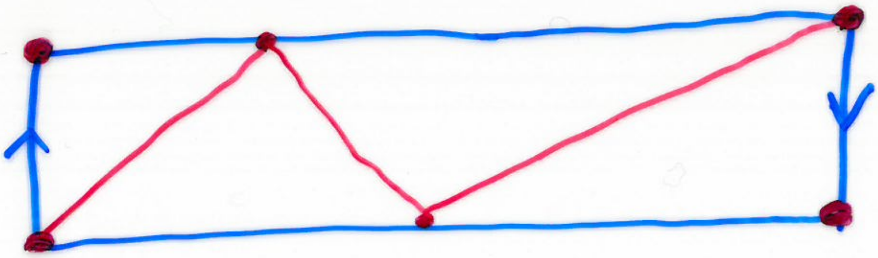
Definition If X is a topological space with triangulation K, $h : |K| \to X$ then we define

$$\chi(X) = \chi(K).$$

Example

$$\chi(\text{torus}) = 9 - 27 + 18 = 0.$$

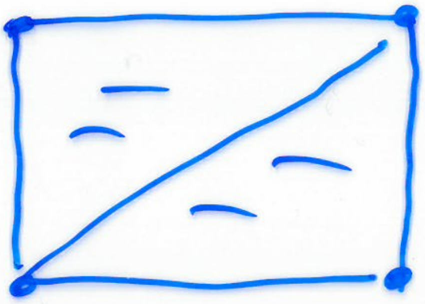
__Example__ Determine the Euler characteristic of the Möbius band



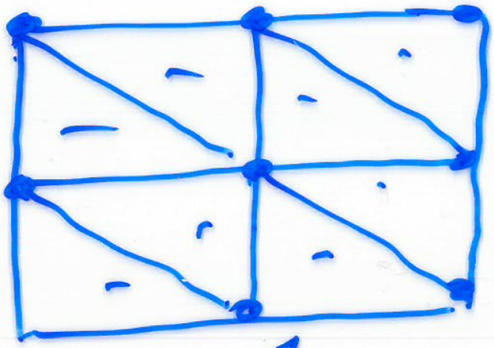
$\chi(\text{Möbius band}) = 4 - 8 + 4 = 0$

Initial attempts are proving Theorem 1 focused on:

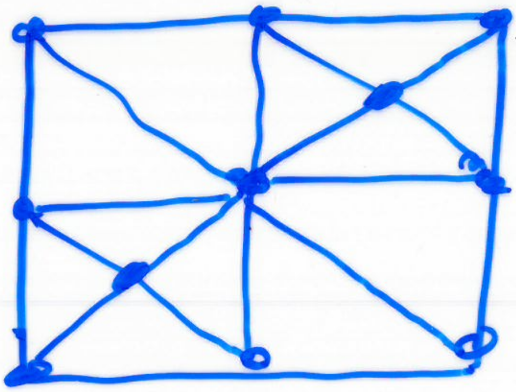
__Hauptvermutung__: If K and L are triangulations of X then there are subdivisions K' of K and L' of L such that $K' = L'$.



K



L



$K' = L'$

The Hauptvermutung was proved for simplicial complexes of dimension ≤ 3 by Morse in the 1950s

In 1961 John Milnor proved the Hauptvermutung false in dimensions ≥ 6.