Data Analysis (Continued)

	H	M	R	C	W
H	0	11	10	14	22
M	11	0	3	13	21
R	10	3	0	12	20
C	14	13	12	0	16
W	22	21	20	16	0

Choose $\varepsilon > 0$.

G_ε is the graph with vertices H, M, R, C, W and an edge

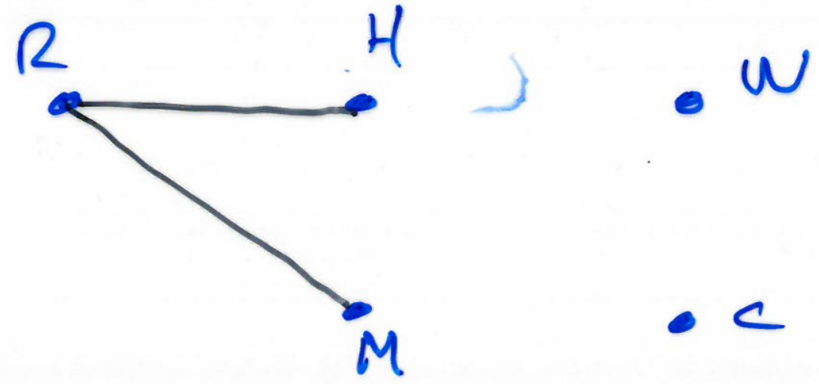
whenever $\mathrm{dist}(A, B) \leq \varepsilon$.

We regard G_ε as a subspace of $\mathbb{E}^5$ consisting of the standard basis vectors

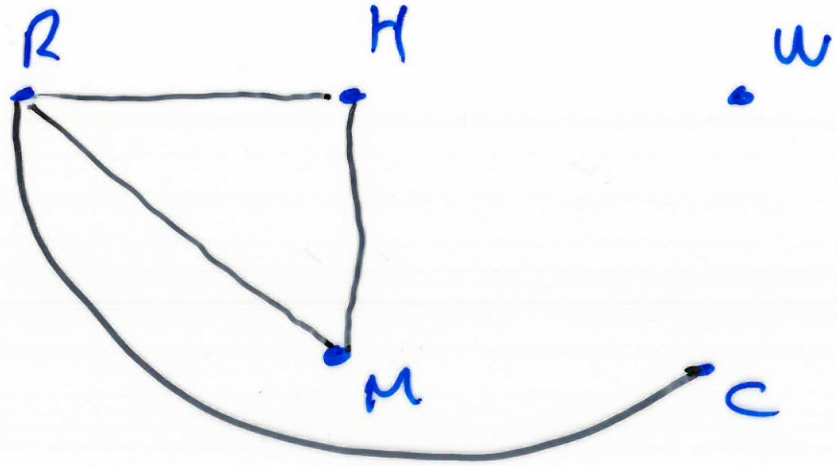
$H = e_1$, $M = e_2$, $R = e_3$, $C = e_4$, $W = e_5$

and the line segment from $A = e_i$ to $B = e_j$ whenever the edge A — B lies in the graph G_Σ.

G_{10}:



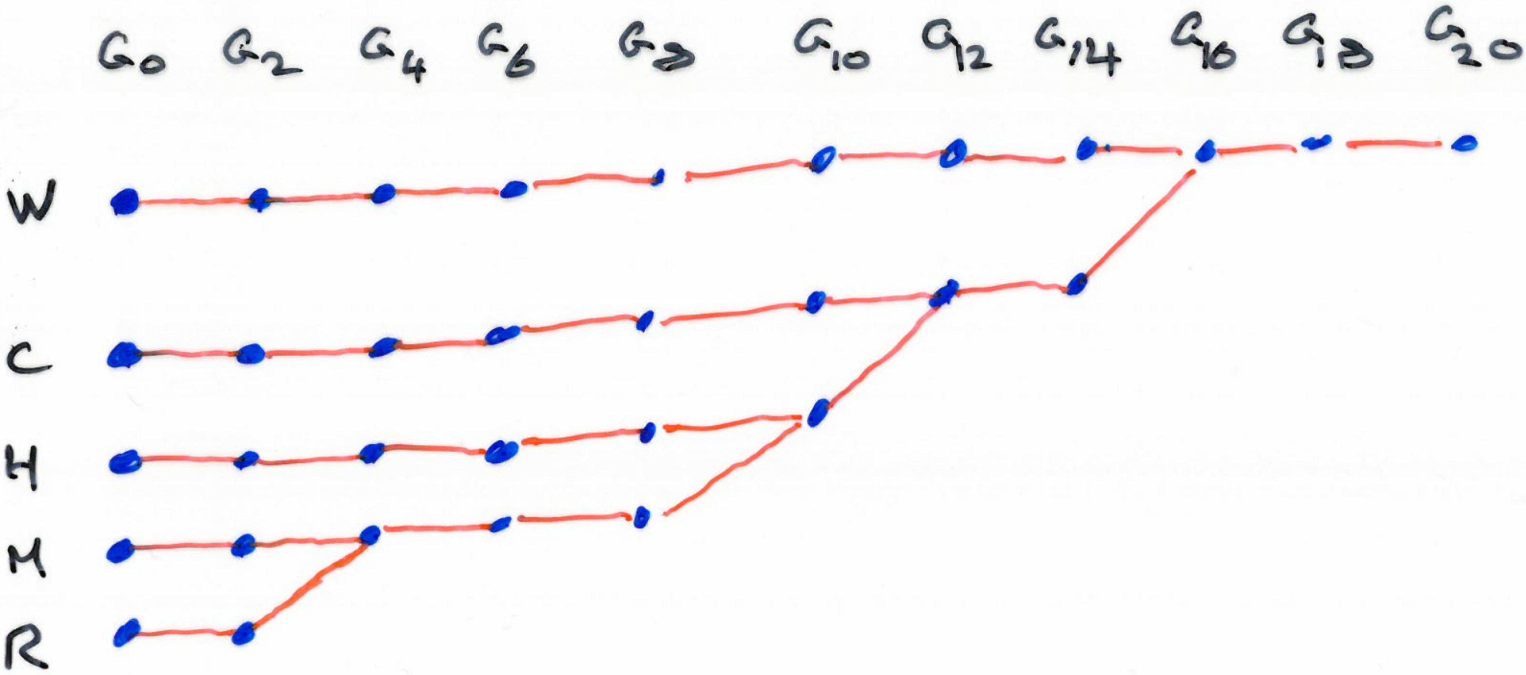
G_{12}:



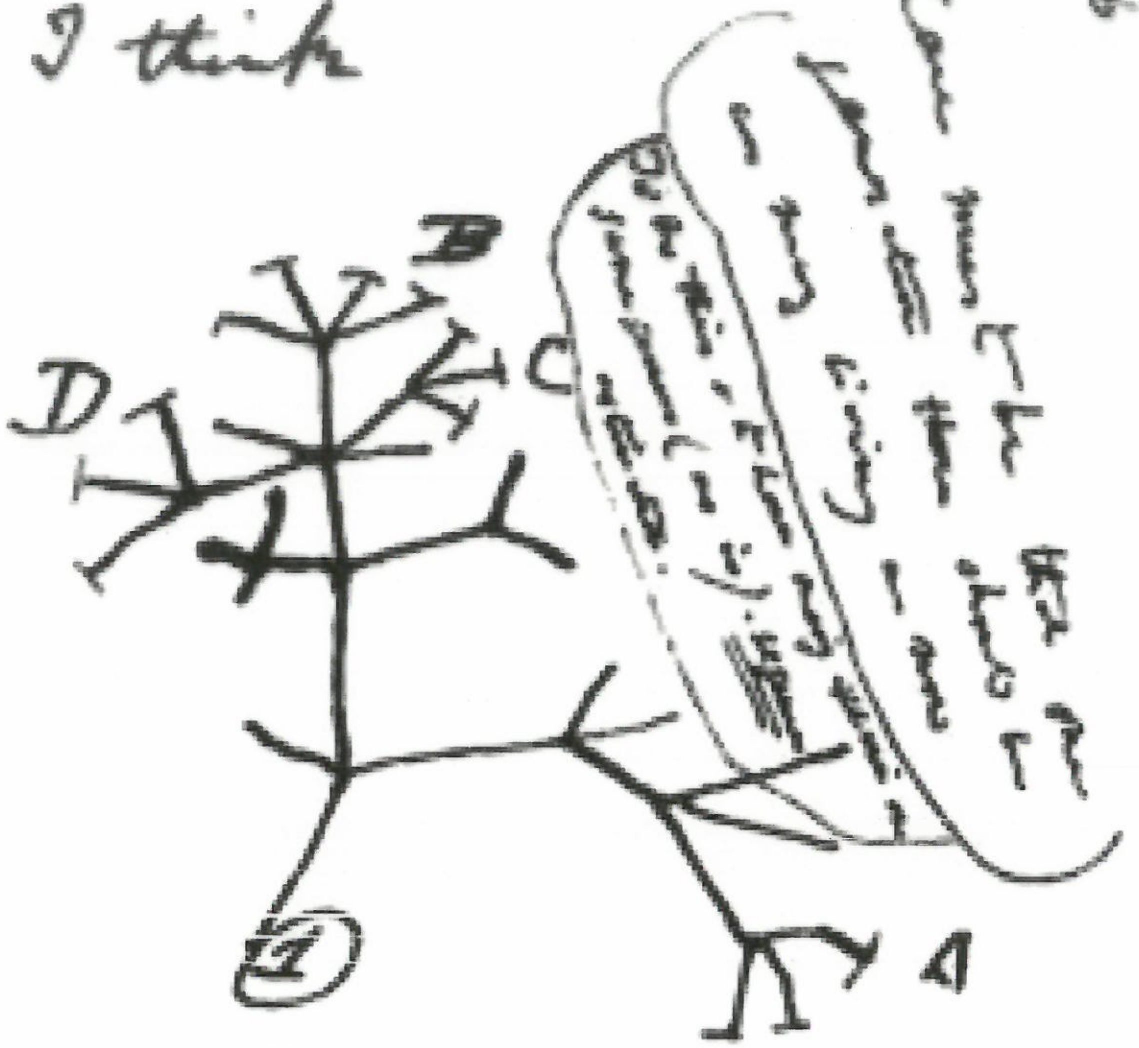
So G_{10} has three connected components X_{HRM}, X_c, X_W say.

And G_{12} has has two connected components X_{RHMC}, X_W.

A dendrogram summarizes the inclusions of connected components.



I think



Thus between A & B. immens gap of relation. C & B. the finest gradation, B & D rather greater distinction. Thus genera would be formed. — bearing relation

Continuity

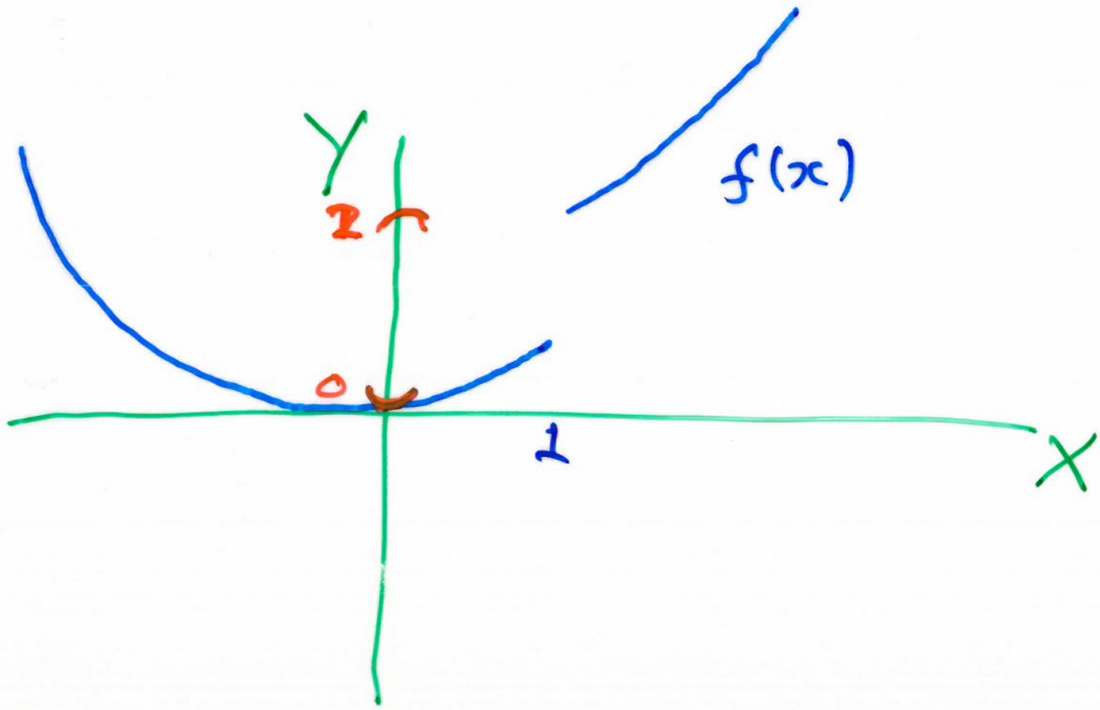
Defn Let X, Y be topological spaces. A function $f : X \to Y$ is <u>continuous</u> if the inverse image of every open set in Y is an open set in X.

We often use the term <u>map</u> to mean a continuous function.

Example $X = \mathbb{E}^1$, $Y = \mathbb{E}^1$.

Consider $f : \mathbb{E}^1 \to \mathbb{E}^1$ given by

$$f(x) = \begin{cases} x^2, & x \le 1 \\ x^2+1, & x > 1 \end{cases}$$



Consider $U = (0,2) \subset Y = \mathbb{E}^1$.

Then $f^{-1}(U) = [-\sqrt{2}, 1]$ is not open in $X = \mathbb{E}^1$.

Hence $f : X \to Y$ is **not** continuous.

<u>Example</u> $X = (-\infty, 1) \cup (1, \infty)$

$Y = \mathbb{R}$

$g : X \to Y$

$$g(x) = \begin{cases} x^2 & x \leqslant 1 \\ x^2+1 & x > 1 \end{cases}$$

This function <u>is</u> continuous.

Major definition:

<u>Defn</u> A continuous function of topological spaces $f : X \to Y$ is a <u>homeomorphism</u> if there exists a continuous function $g : Y \to X$ such that

$$g(f(x)) = x \quad \text{for all } x \in X$$

and

$$f(g(y)) = y \quad \text{for all } y \in Y.$$

If $f: X \to Y$ is a homeomorphism then we say that X is <u>homeomorphic</u> to Y.