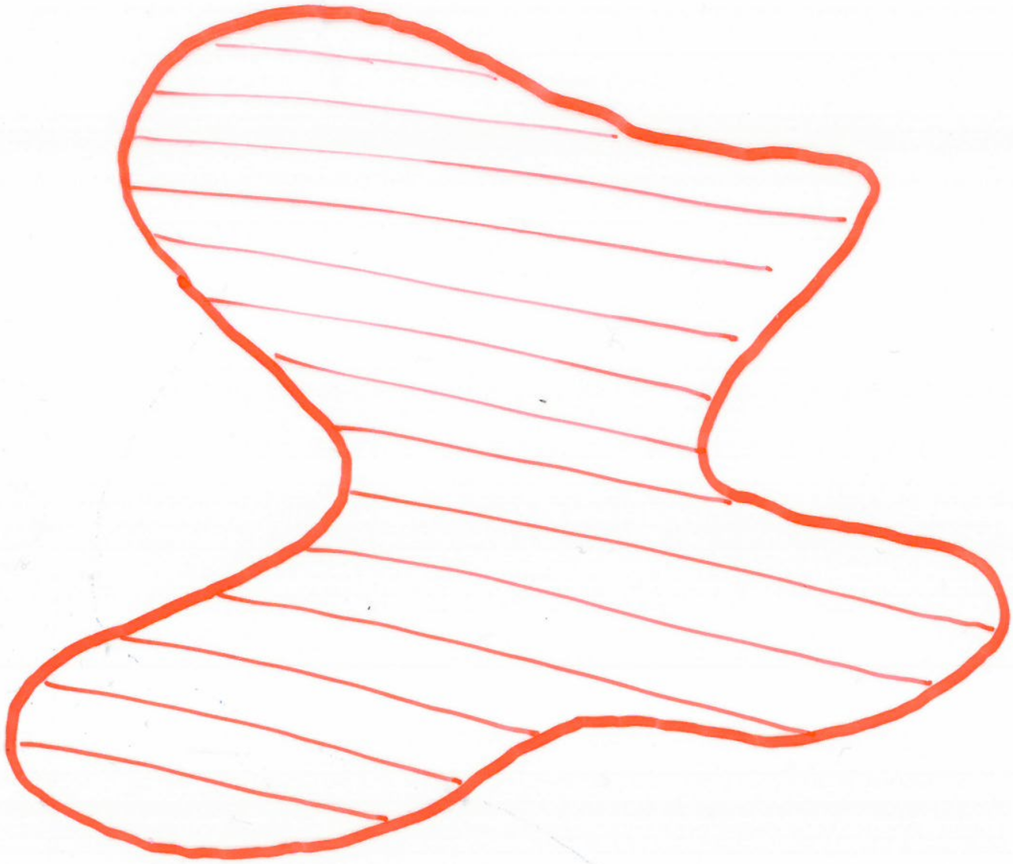
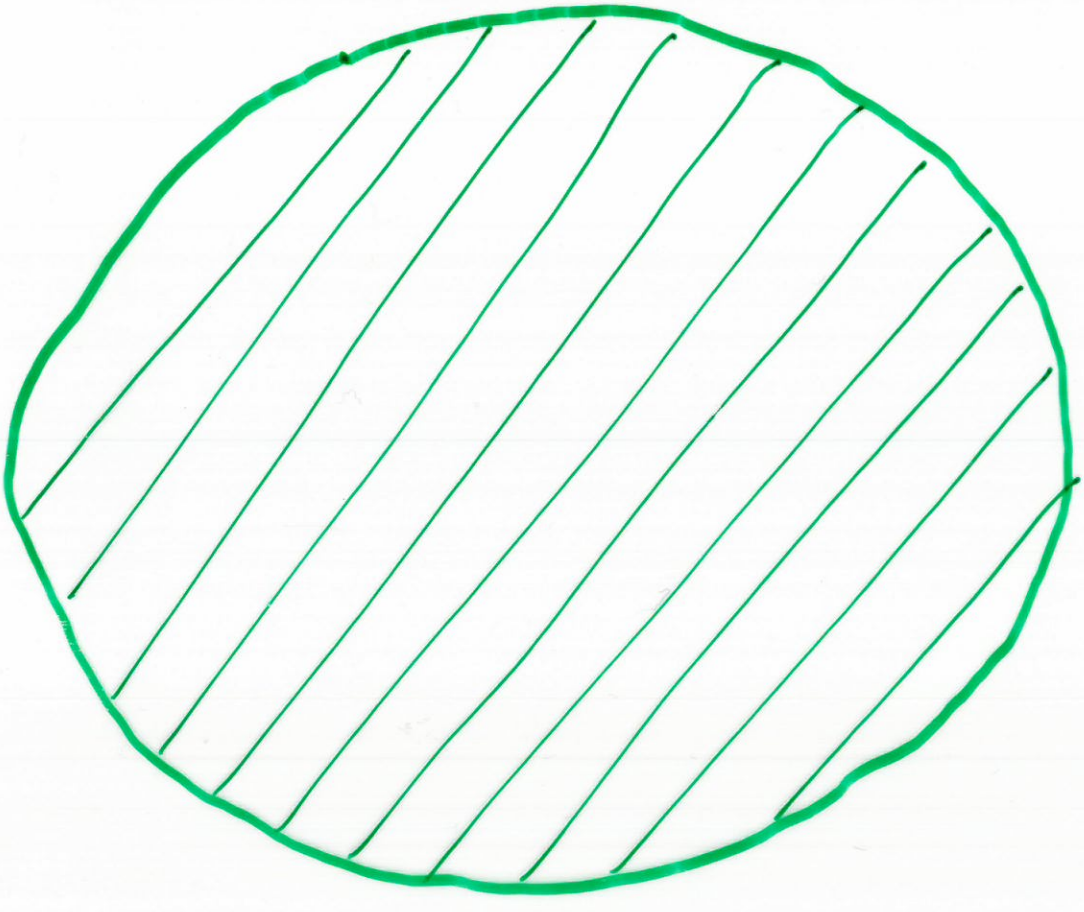
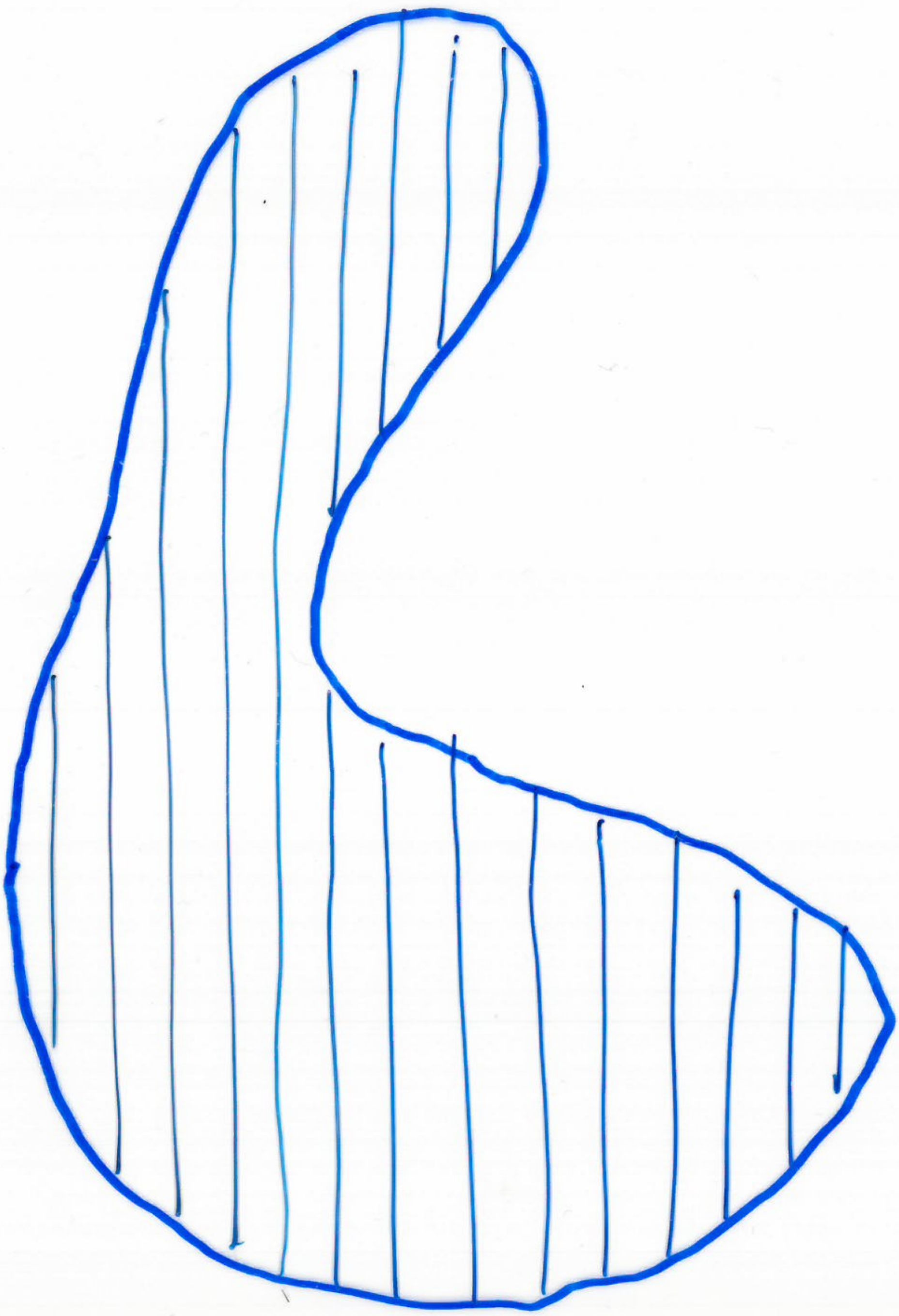


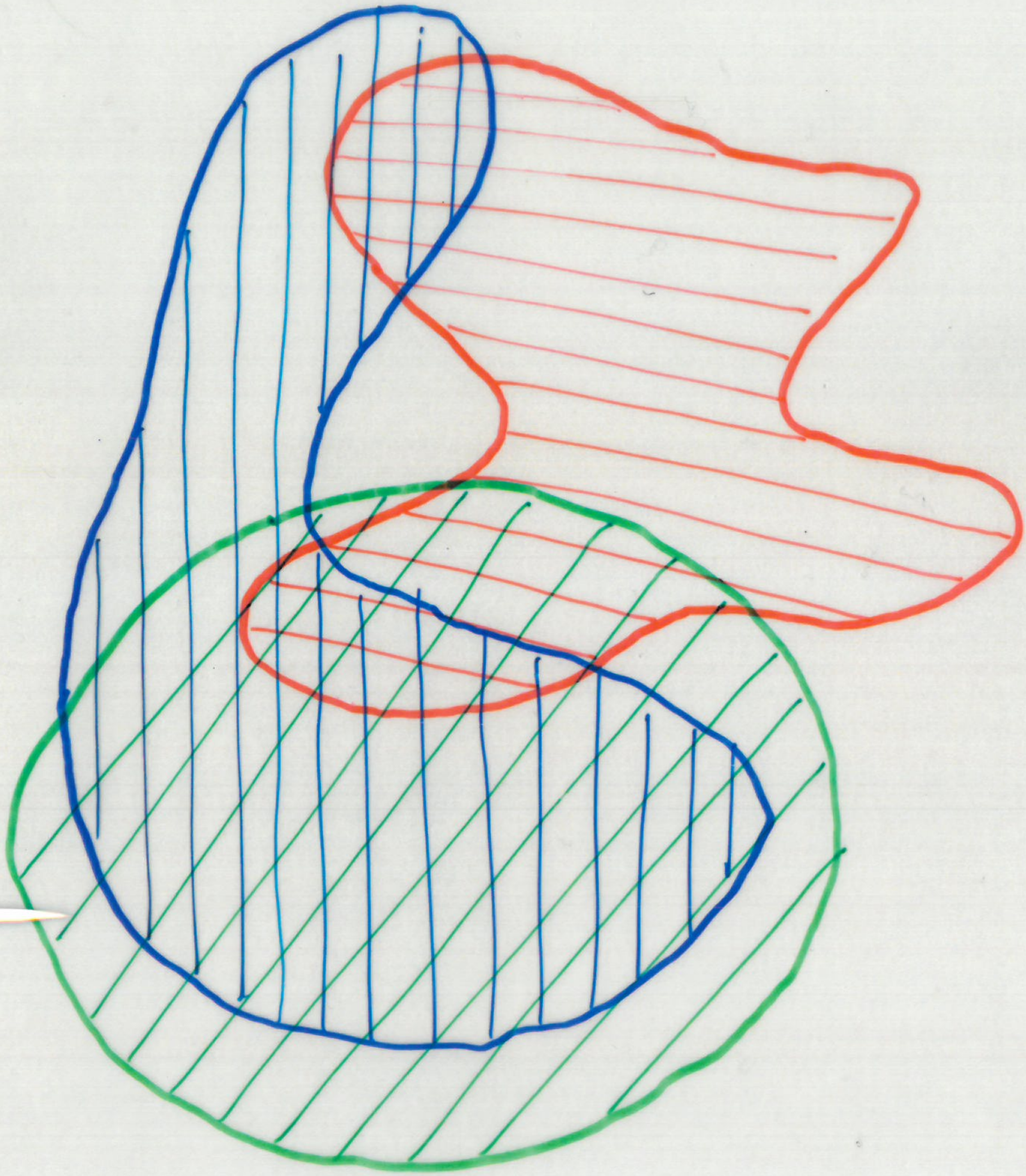
U_1



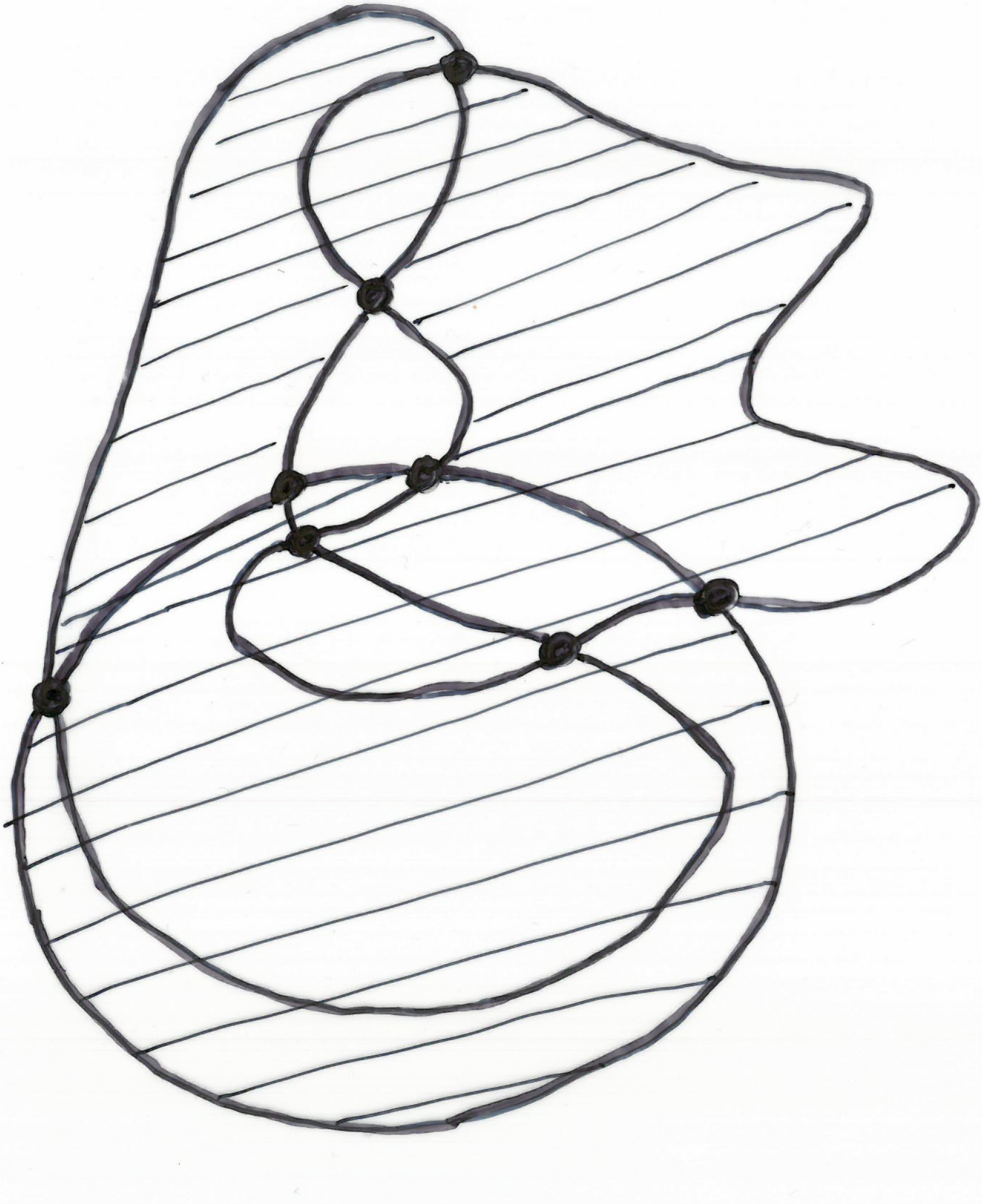
u_2



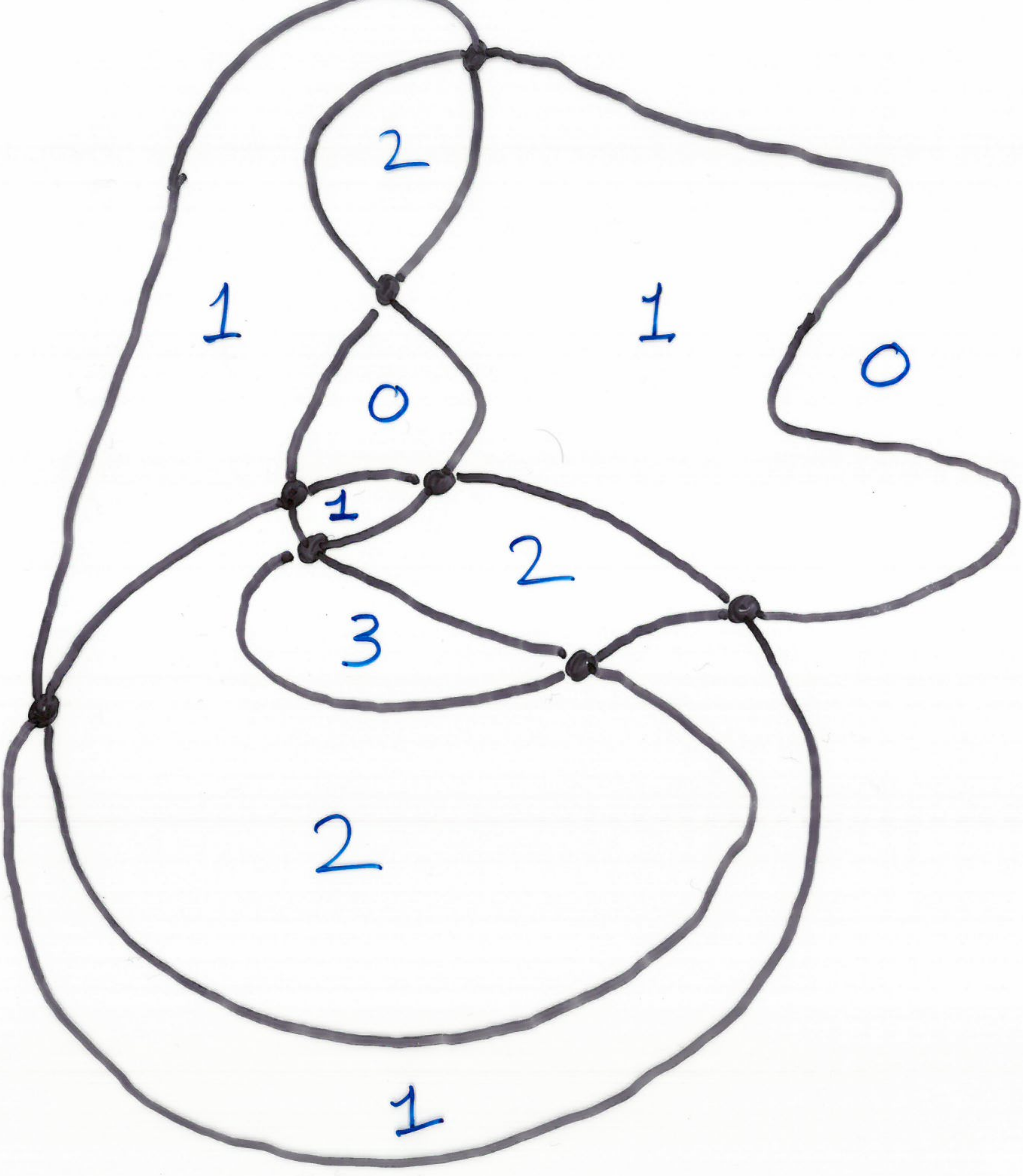
U_3



$X = U_1 \cup U_2 \cup U_3$



$X = U_1 \cup U_2 \cup U_3$



$\omega(x) = |\{i : x \in U_i\}|$

$$\int_X \omega \, dx = (6 \times 2 + 2 \times 3) - (6 \times 1 + 8 \times 2 + 2 \times 3) + (4 \times 1 + 3 \times 2 + 3)$$

$$= 18 - 28 + 13$$

$$= 3$$

<u>Theorem</u> Let $X \subseteq \mathbb{R}^2$ be a region with subregions $U_1, U_2, \ldots, U_t \subseteq \mathbb{R}^2$ such that

$$X = U_1 \cup U_2 \cup \cdots \cup U_t .$$

Let $\omega : X \longrightarrow \mathbb{N}$ be the weight function

$$\omega(x) = |\{ i : x \in U_i \}| .$$

Suppose that each U_i has the same Euler characteristic

$$\chi(U_i) = c \quad \text{say.}$$

Then

$$t = \frac{1}{c} \int_X \omega \, dx$$

<u>Proof</u> Let

$$\mathbb{1}_{U_i} : X \to \mathbb{N}$$

be defined as

$$\mathbb{1}_{U_i}(x) = \begin{cases} 1 & \text{if } x \text{ is in } U_i \\ 0 & \text{otherwise} \end{cases}$$

$$\int_X \omega \, dx = \int_X \Big(\sum_{1 \le i \le t} \mathbb{1}_{U_i} \Big) dx$$

$$= \sum_{1 \le i \le t} \int_X \mathbb{1}_{U_i} \, dx$$

think

$$= \sum_{1 \le i \le t} \chi(U_i) = tc \, .$$

QED

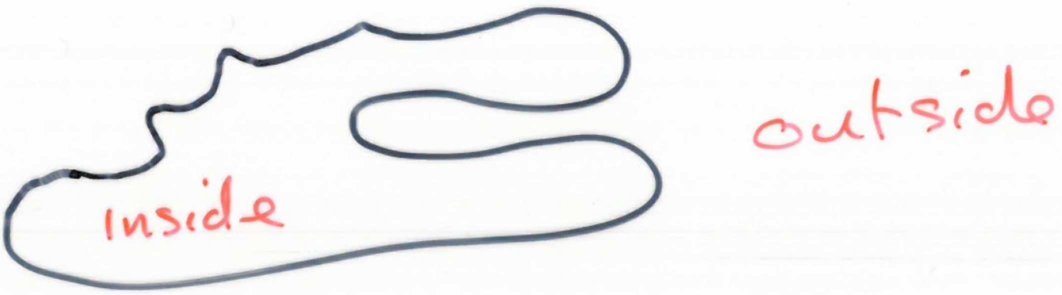
Closer look at Euler Characteristics

Our proof of $\chi(S^2) = 2$

football

Mars

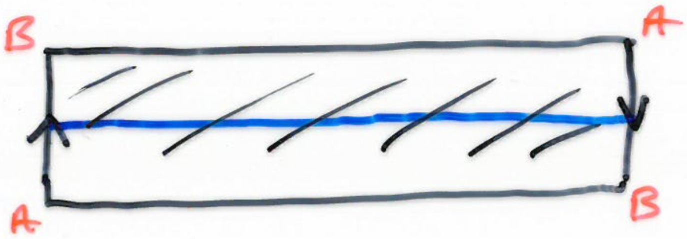
used the fact that any loop in the plane with no self intersections



has an inside and and outside, i.e. the loop cuts the plane into two regions.

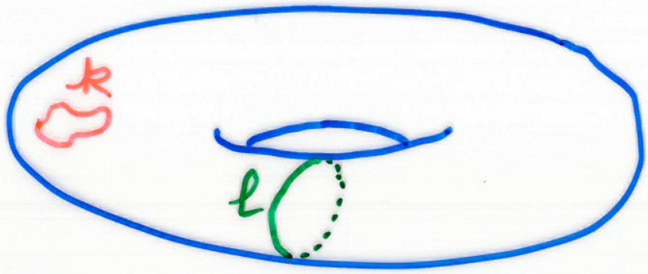
Is this obvious?

Example Consider the Möbius strip.



Draw a loop around the centre of the strip. This loop does **not** cut the strip into two pieces.

Example consider a torus



Loop k divides the torus into two pieces. But loop l does not!!

Example Is it obvious that the following loop in the plane has an inside and an outside?