E

G

J L

L

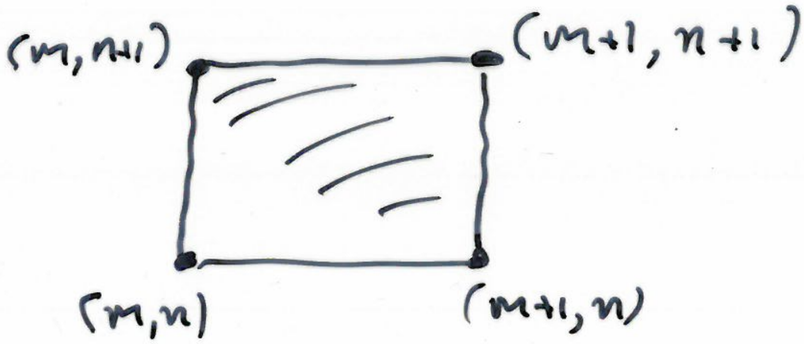
I S

How many objects are there?
letters

The above image represents a region $X \subseteq \mathbb{R}^2$ where X is a union of unit squares

$$[m, m+1] \times [n, n+1]$$

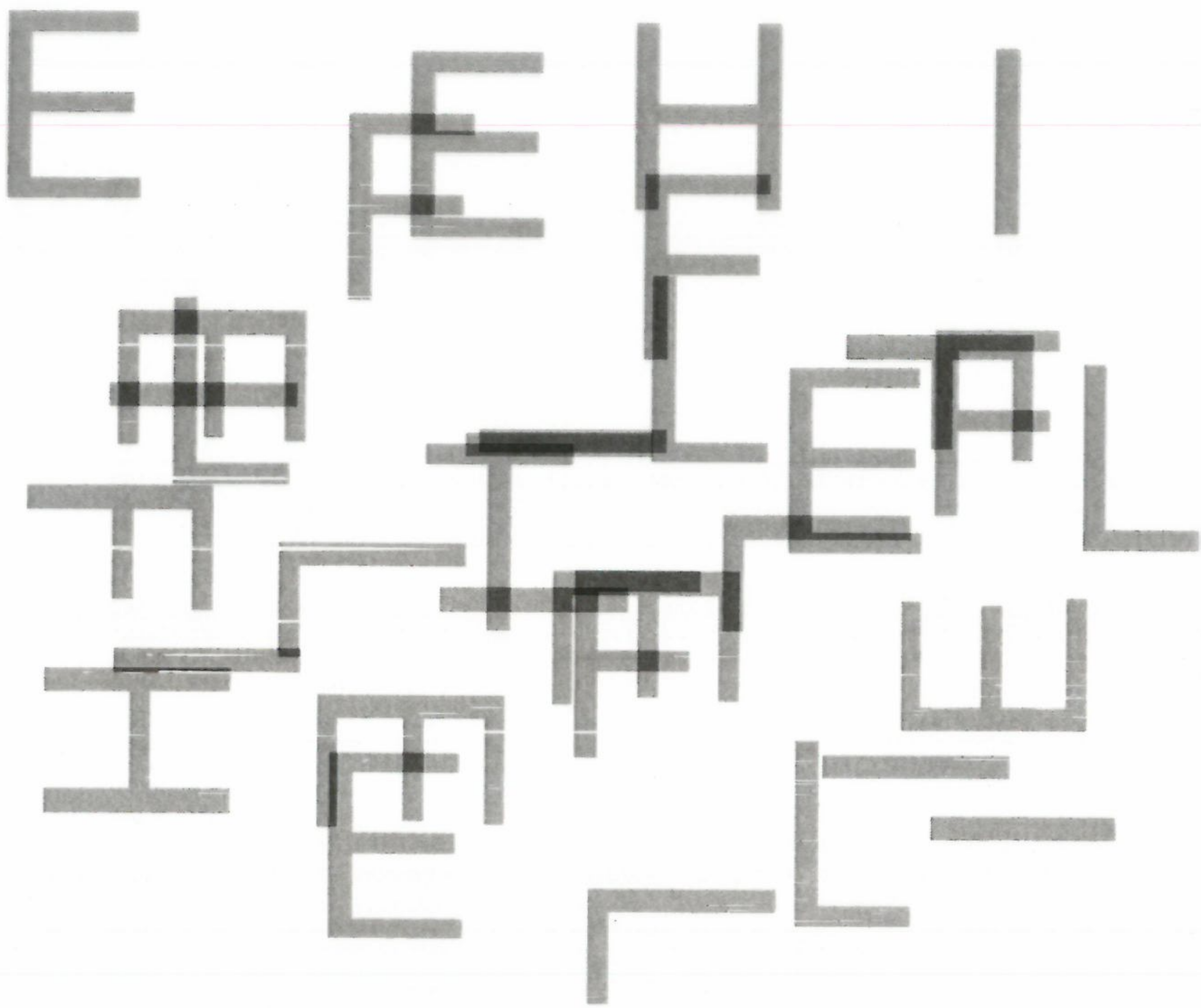
for various integers m, n.



Each letter in the image is a black region with no holes.

From last lecture:

$$\text{number of letters} = \chi(X) = V - E + F$$



Third problem

How many letters are in the above picture? And how could we count them on a computer?

We can't use $\chi(X)$ since some letters overlap.

Alternative version of Third problem

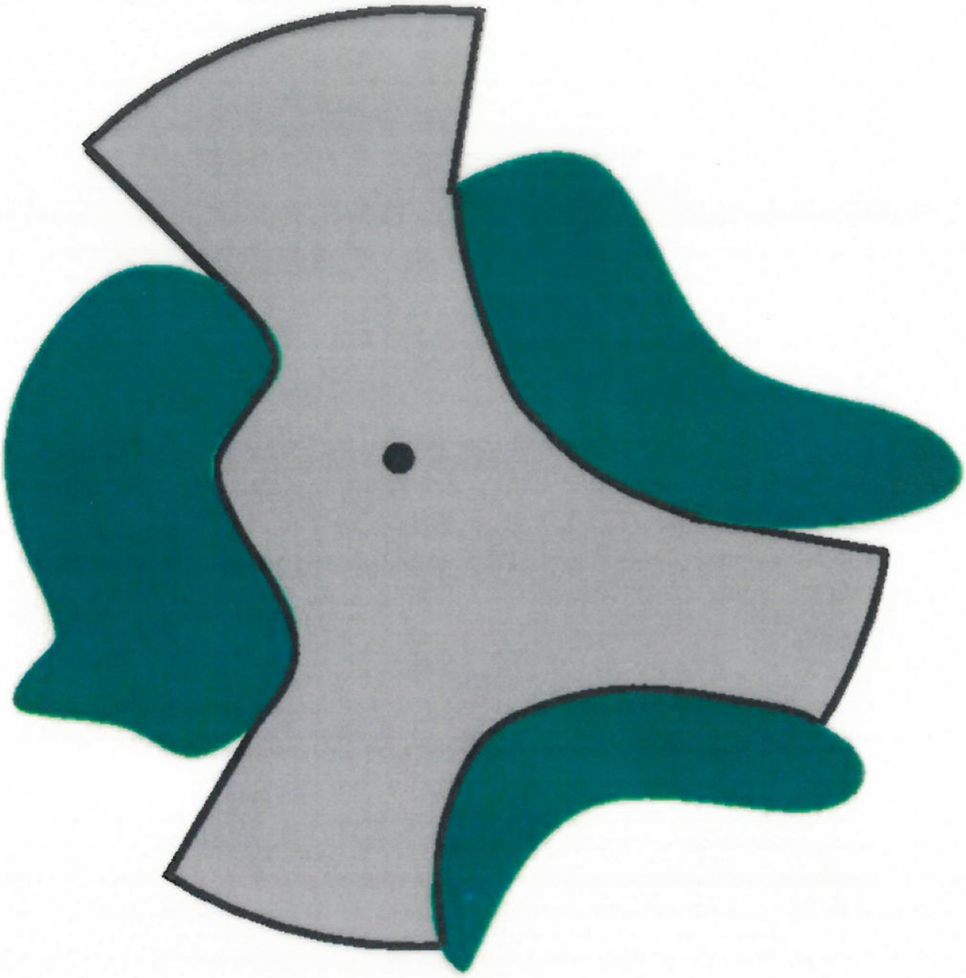
(see recent paper by Yuliy Baryshnikov and Robert Ghrist)

A Texas farmer invests in thousands of cheap sensors

The sensors are spread out in an array / grid covering the ranch.

When activated from the farmhouse a sensor counts the number of cows in the line-of sight region around the sensor and returns this number, together with its sensor id to the farm.

How can the farmer determine the number of cows on the ranch from this data?



The grey visibility area
of a black point cow, partially
hidden by some green objects.

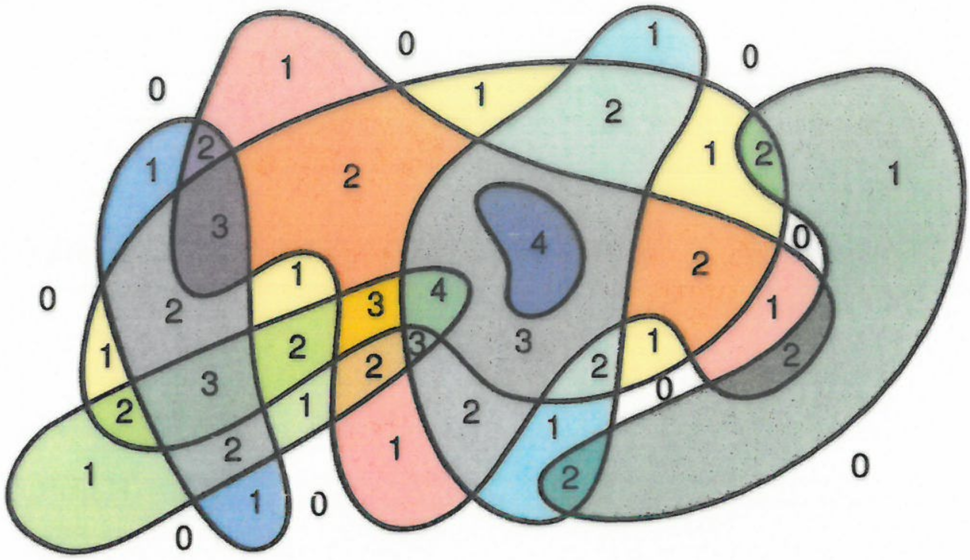


FIG. 4.1. *A collection of contractible patches $\{U_\alpha\}$ in $\mathbb{R}^2$ corresponding to the supports or 'visibility regions' of seven targets. The collection decomposes $\mathbb{R}^2$ into cells labeled according to the height function h returned by a dense sensor network.*

A collection of visibility regions of cows. Each visibility region has Euler characteristic = 1.

Given a region

$\mathbb{R}^2 \supseteq X$ = union of unit squares

then a weight function

$$\omega : X \longrightarrow \mathbb{N}$$

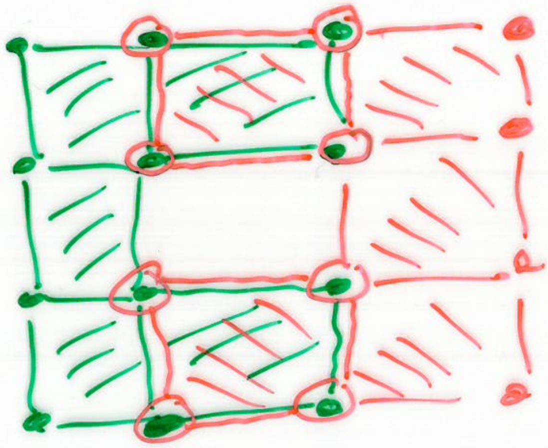
assigns a number

$\omega(v) \in \mathbb{N}$ to each vertex of X

$\omega(e) \in \mathbb{N}$ " " edge " X

$\omega(f) \in \mathbb{N}$ " " face " X

Example 1

$X =$ 

If a vertex, edge or face x involves one colour define

$$\omega(x) = 1.$$

If it has two colours define

$$\omega(x) = 2.$$

Then ω is an example of a weight function.

<u>Defn</u> Given a region $X \subseteq \mathbb{R}^2$ and a weight function

$$\omega : X \longrightarrow \mathbb{N}$$

we define the <u>Euler integral</u> to be

$$\int_X \omega \, d\chi = \sum_v \omega(v) - \sum_e \omega(e) + \sum_f \omega(f)$$

where v, e, f range over vertices and faces of X.

Example 1 (continued)

$$\int_X \omega \, dx = (8 + 8\times 2) - (16 + 8\times 2) + (6 + 2\times 2) = 2$$

[Note: X is a union of two regions, a red one and a green one. This is no accident!]

Theorem

Let $X \subseteq \mathbb{R}^2$ be a region with subregions $U_1, U_2, \dots, U_t \subseteq \mathbb{R}^2$ such that

$$X = U_1 \cup U_2 \cup \cdots \cup U_t .$$

Let $\omega : X \longrightarrow \mathbb{N} = \{0, 1, \dots\}$ be the weight function

$$\omega(x) = |\{ i : x \in U_i \}| .$$

Suppose that each U_i has the same Euler characteristic $\chi(U_i) = c \neq 0$ say.

Then

$$t = \frac{1}{c} \int_X \omega \, d\chi .$$