<u>MA342 web page</u>

Link from my homepage

Link from Blackboard

Text: first 5 chapters of "Basic Topology" by M.A. Armstrong

Tutorials : Tue 1, AC214

Wed 2, AC201

Continuous assessment : 3 in-class tests worth 10% each

Final exam : worth 70%

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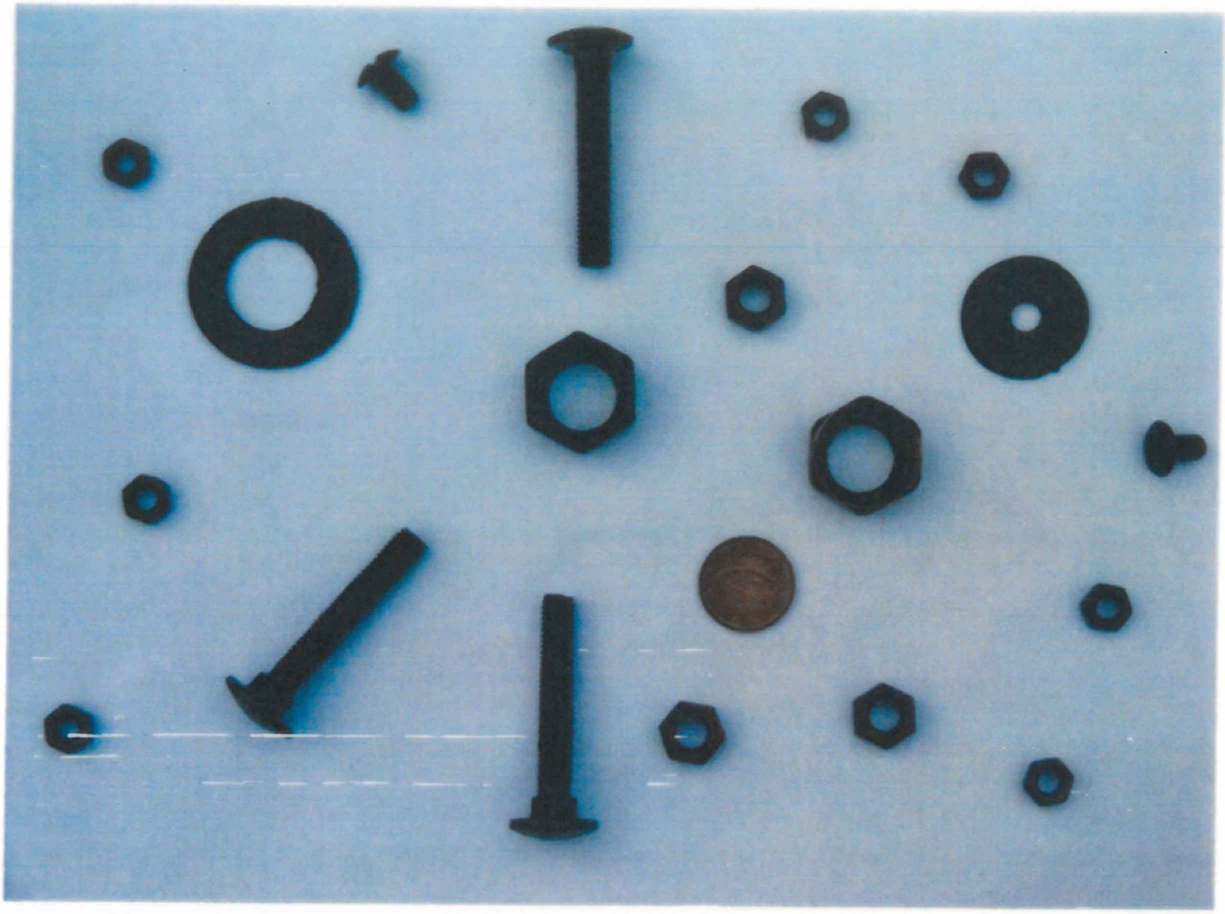
A second problem (easy version)

How many objects are there in the following digital image that don't have hole?



A colour digital image is an array of $m \times n$ pixels; each pixel has a colour and intensity determined by a triple of numbers (r, g, b) where

$0 \le r, g, b \le 255$.

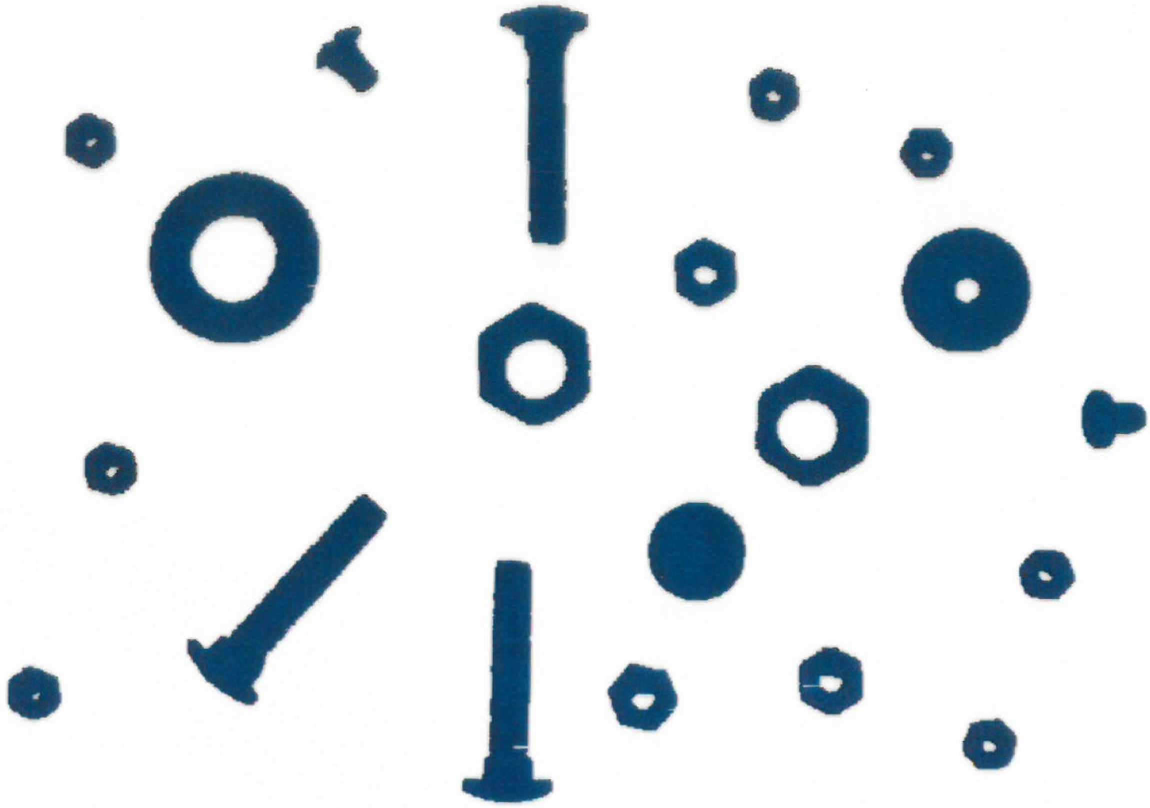
Black is $r=0, g=0, b=0$.

Mathematically a digital image is thought of as a function

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3, \quad (x,y) \longmapsto (r,g,b)$$

Given an image f we can choose some number $T \in \mathbb{R}$ and consider the <u>level set</u>

$$X = \left\{ (x,y) \in \mathbb{R}^2 : \begin{array}{l} f(x,y) = (r,g,b) \\ \text{with } r+g+b \le T \end{array} \right\}$$



In our example the level set X retains enough information to see that there are 20 objects, 6 of which don't have holes.

<u>Second problem</u> (harder version)

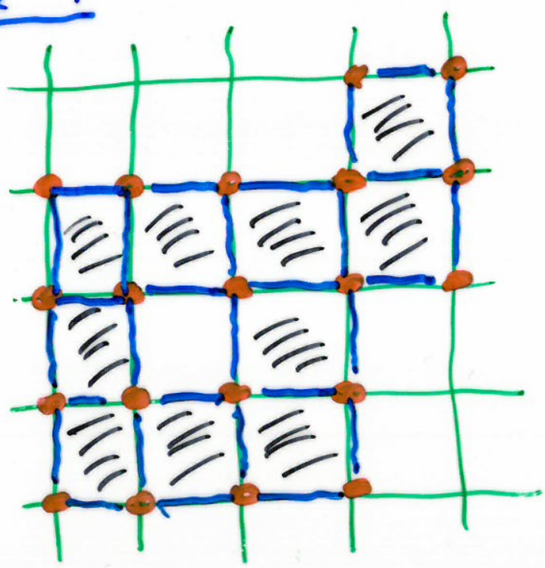
How can we use a computer to determine, from the level set X, the number of objects without holes?

Think of a black pixel as a shaded square with four edges of length 1 and four integer vertices.

Let us call a union of such squares a <u>region</u>.

<u>Example 1</u>

A region $X =$



<u>Definition</u> For a region $X \subseteq \mathbb{R}^2$ define the <u>Euler</u> <u>characteristic</u>

$$\chi(X) = V - E + F$$

where V, E, F are the numbers of vertices, edges and faces.

Example 1 (continued)

$$\chi(X) = 20 - 30 + 10 = 0$$

Proposition 1 Let $X \subseteq \mathbb{R}^2$ be a region containing no holes. (A coin or bolt!) Then

$$\chi(X) = 1$$

Proof Think of X as a collection of black fields on Mars, the complement being one white field.

Then

$$\chi(\text{Mars}) = 2 = \chi(X) + 1$$

So $\chi(X) = 1$.

QED

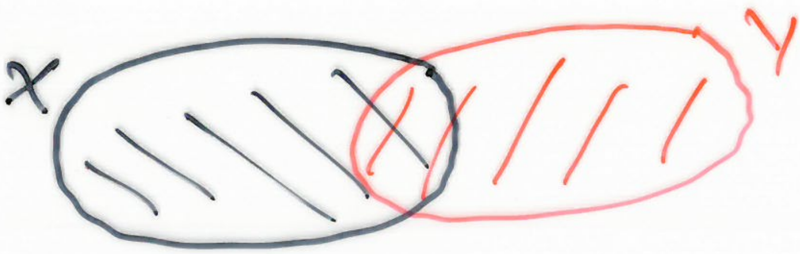
Proposition 2 Let $X \subseteq \mathbb{R}^2$ be a region containing k holes. Then $\chi(X) = 1-k$.

Proof X is got by removing k faces/fields from a no hole region.

Proposition 3 Let $X, Y \subseteq \mathbb{R}^2$ be two regions. Then

$$\chi(X \cup Y) = \chi(X) \cup \chi(Y) - \chi(X \cap Y)$$

Proof



Example For the level set of our original example

$$\chi(X) = \text{number of objects without holes.}$$

But $\chi(X) = V - E + F$ is easily computed on a computer.