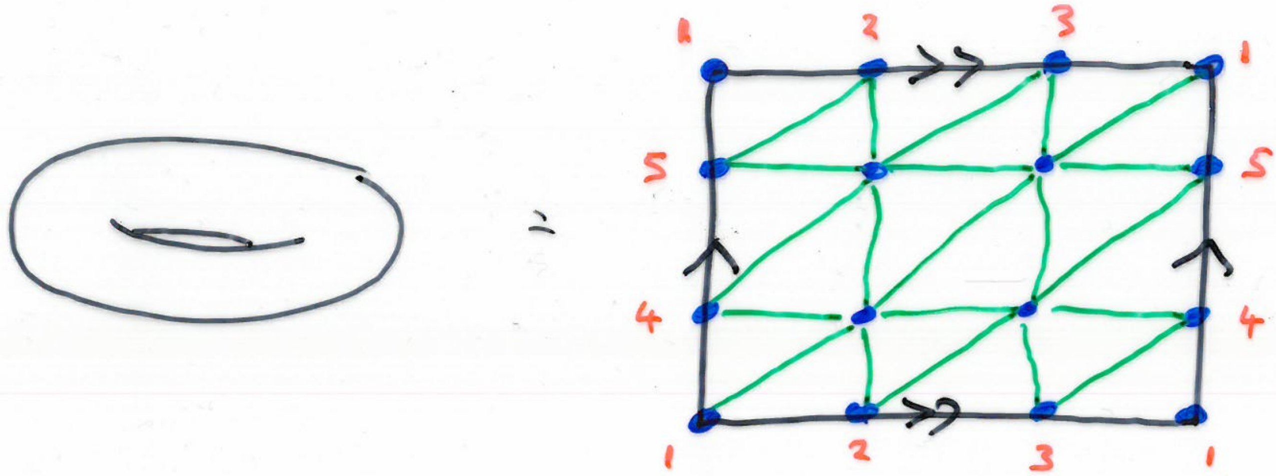
Example Triangulation of the torus:



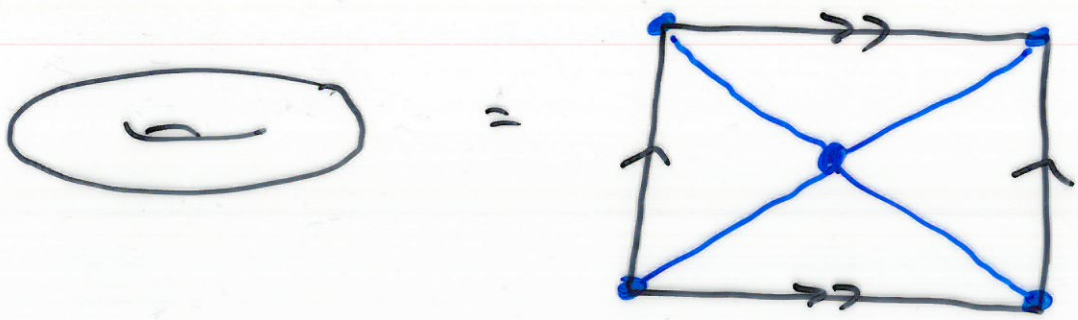
α_k = number of k-simplices

$\alpha_0 = 9$

$\alpha_1 = 27$

$\alpha_2 = 18$

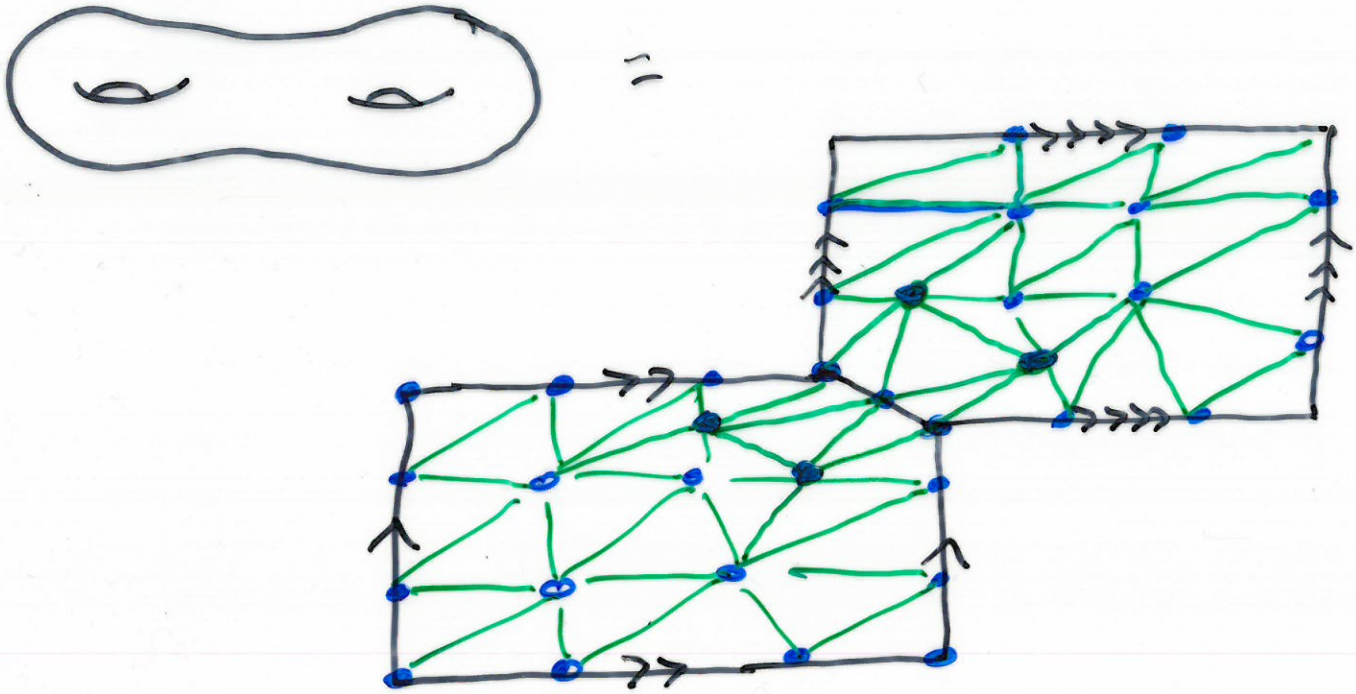
Example (non-example)



Not a triangulation

A k-simplex must have $k+1$ distinct vertices.

<u>Example</u> Triangulation of "double torus"



$\alpha_0 =$

$\alpha_1 =$ exercise.

$\alpha_2 =$

<u>Definition</u> Let K be a simplicial complex with α_k k-simplices. The <u>Euler characteristic</u> of K is

$$\chi(K) = \alpha_0 - \alpha_1 + \alpha_2 - \alpha_3 + \cdots$$

Theorem 1 If simplicial complexes K, L are such that $|K|$ is homeomorphic to $|L|$ then

$$\chi(K) = \chi(L).$$

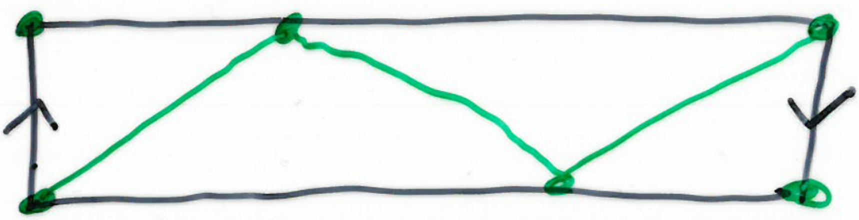
Defn If X is a topological space with triangulation K, $h : |K| \to X$ then we define

$$\chi(X) = \chi(K).$$

Example

$$\chi(\text{torus}) = 9 - 27 + 18 = 0$$

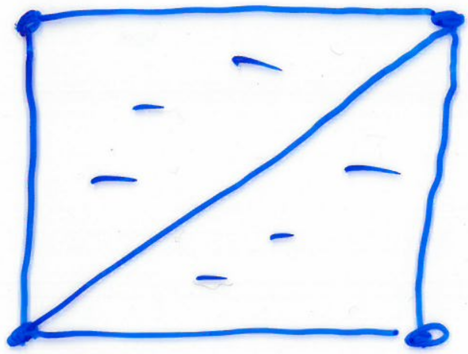
Example Determine the Euler characteristic of the Möbius band



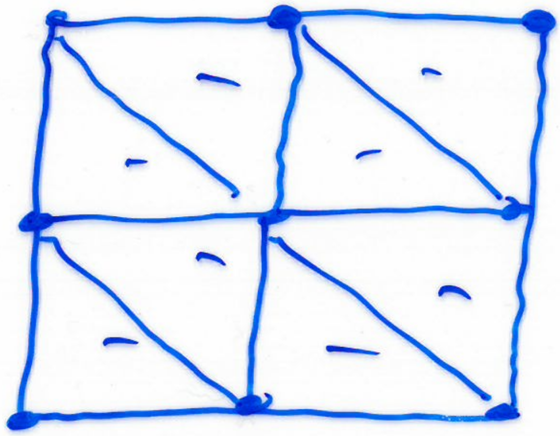
$\chi(\text{Möbius band}) = 4 - 8 + 4 = 0$

Initial attempts at proving Theorem 1 focused on the:

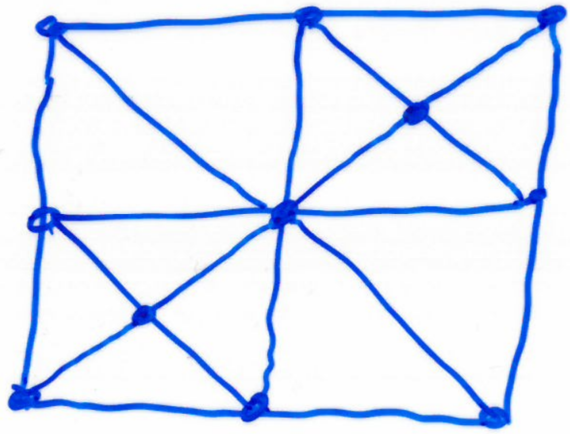
Hauptvermutung: If K and L are triangulations of X then there are subdivisions K' of K and L' of L such that $K' = L'$.



K



L



$L' = K'$

The Hauptvermutung was proved for simplicial complexes of dimension ≤ 3 by Morse in 1950s.

In 1961 John Milnor proved the Hauptvermutung **false** in dimensions ≥ 6.