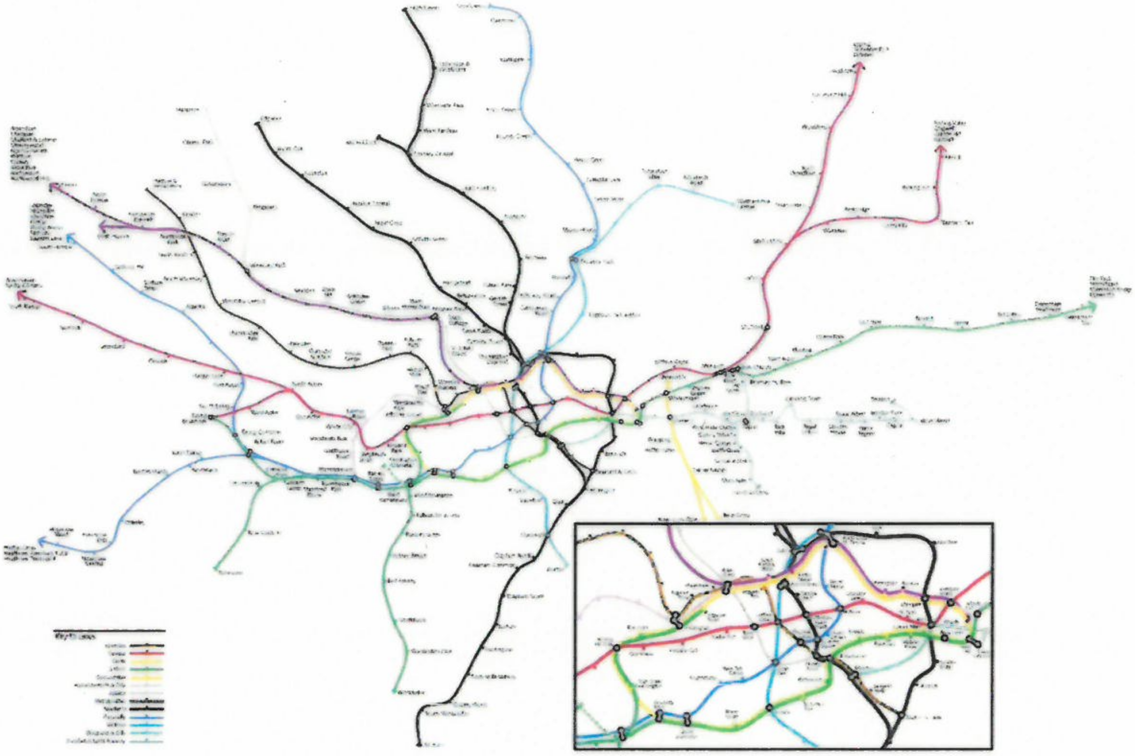
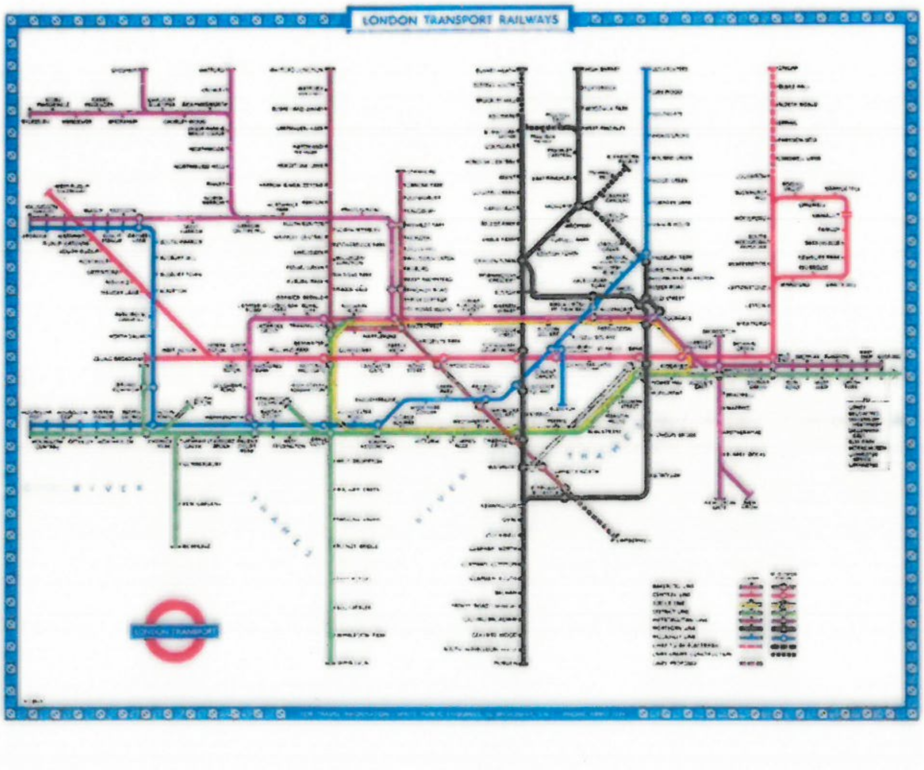
Topology MA 342

Graham Ellis

<u>Topology</u> studies properties of spaces or shapes that remain unchanged under continuous deformations such as bending and stretching (but <u>not</u> tearing or gluing)

We use topology when planning a journey on any metro.




LONDON TRANSPORT RAILWAYS

The usual map of the London Underground is a continuous deformation of a geographical map.

The geographical map tells us the distance between two stations whereas the usual deformed map does not. But the deformed map retains enough properties for us to plan our journey

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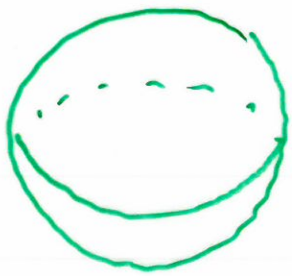
Top- ~ place (as in topography)

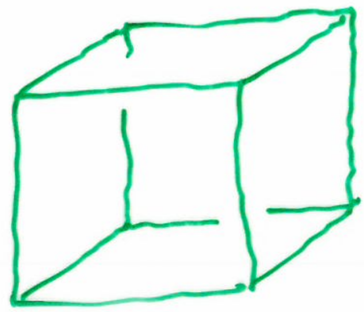
-ology ~ study (as in biology)

Motivations for topology:

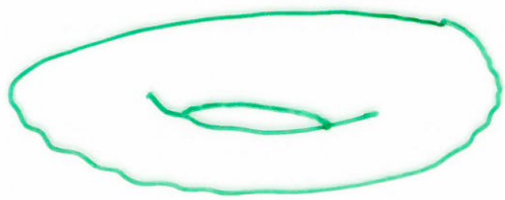
Analysis ~ how can we best study and generalise "continuity"

Geometry ~ what properties are common to a hollow sphere and hollow cube





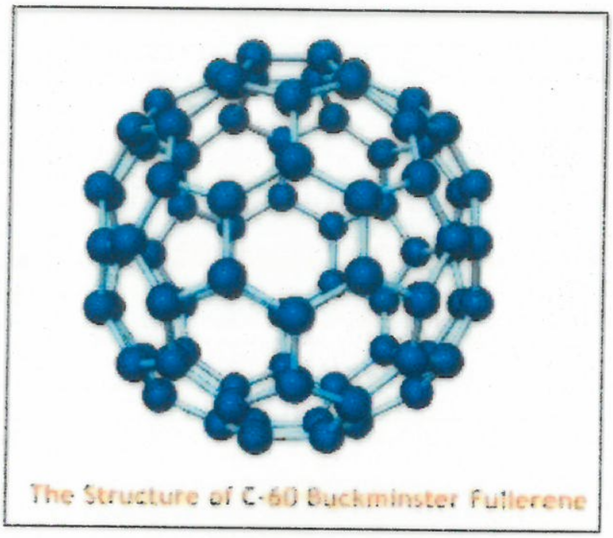
but don't hold for a hollow torus?

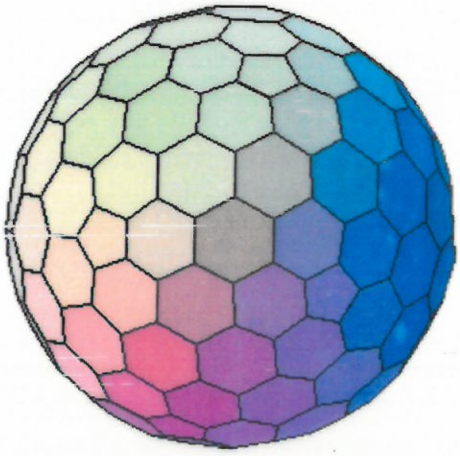


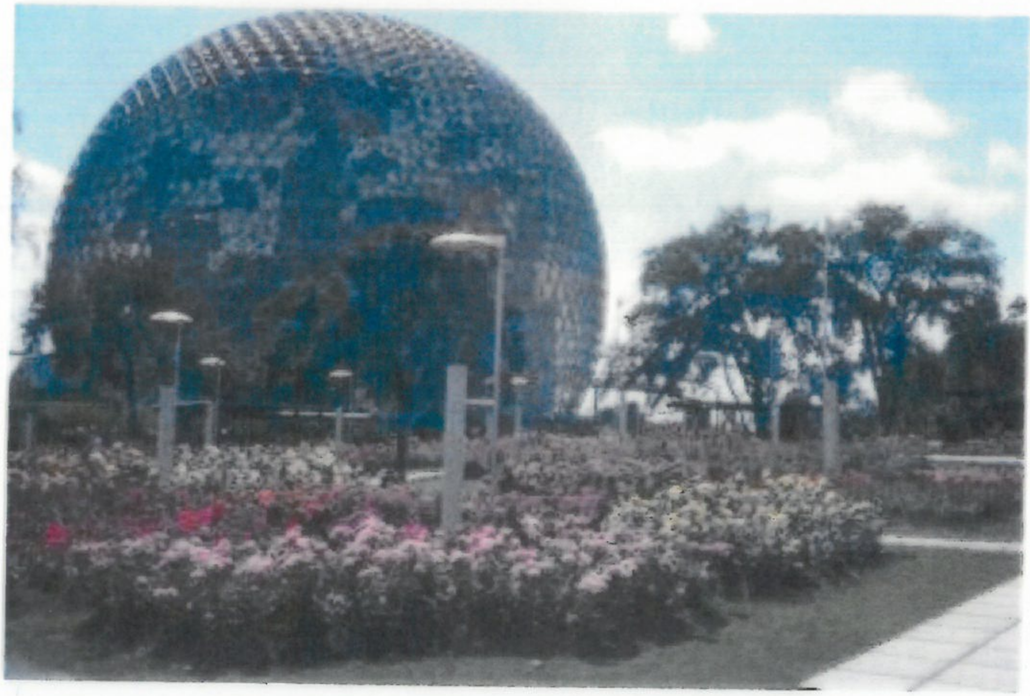
A first Problem

How many pentagons are there in a

- Soccer ball
- a carbon-60 molecule (found in soot in your chimney)
- other potential carbon fullerenes (The 1996 Nobel Prize for chemistry was awarded for the discovery of such carbon molecules)
- a Buckminster dome ?





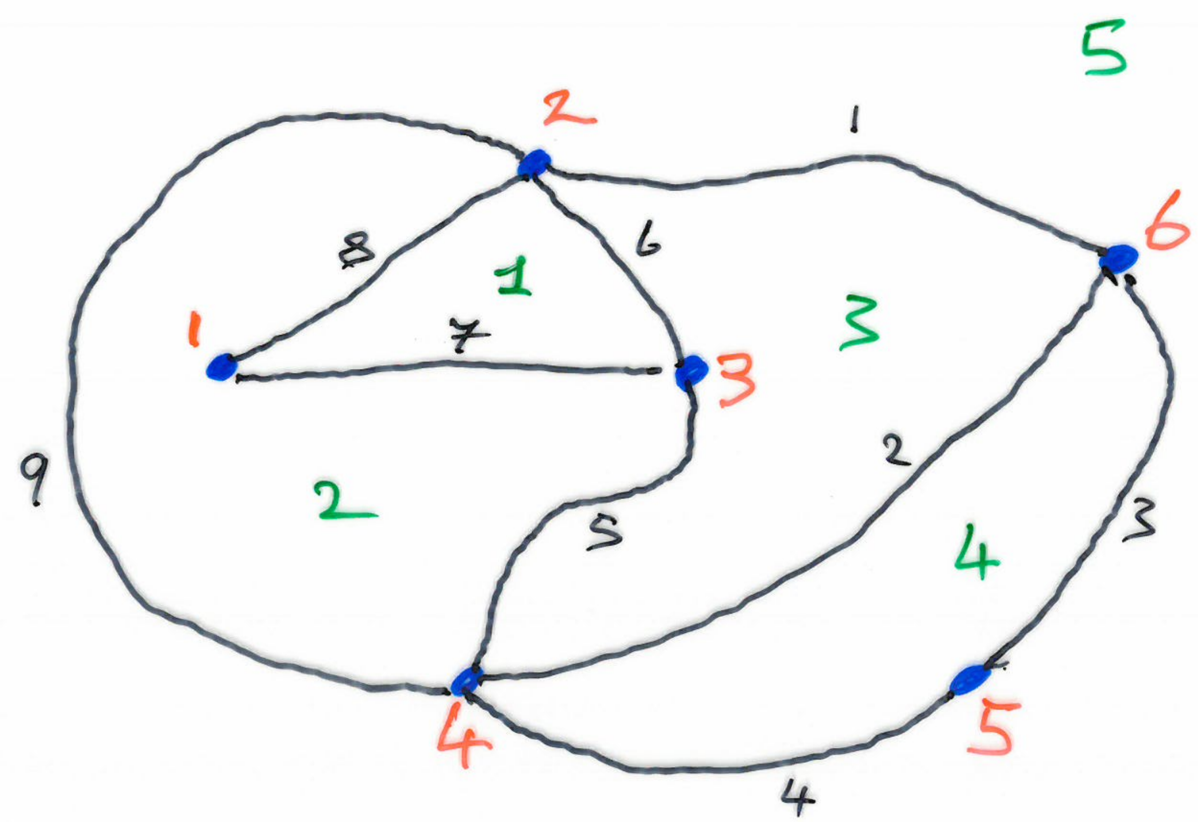


How many pentagons in a soccer ball ?

Let's design an arbitrary system of villages and expressways on Mars, subject to:

i) people can drive between any two villages

ii) there are no bridges or tunnels and expressways meet only at villages.



$V = 6$

$E = 9$ $V - E + F = 6 - 9 + 5 = 2$

$F = 5$

why do we always get

$V - E + F = 2$?

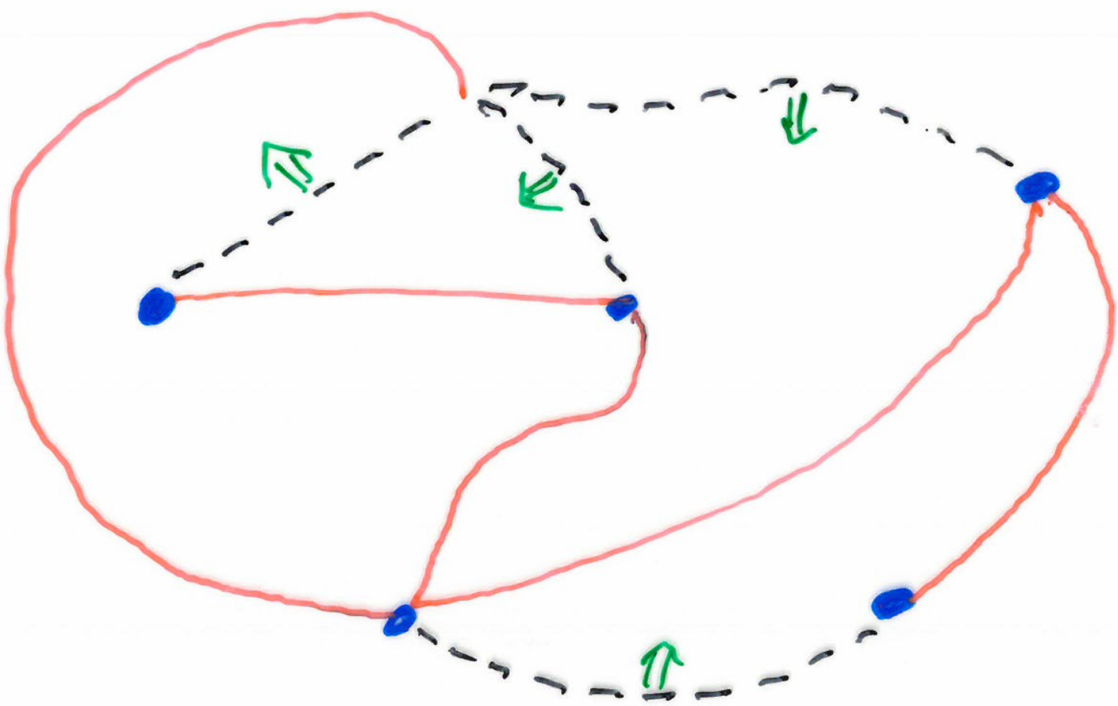
Choose a subsystem T of expressways

such that

i) there are no loops

ii) we can still travel between

any two villages.



Each black dotted expressway joins two villages in the system T and thus determines a unique loop, all but one of whose expressways are red.

The inside of this loop is a field. (So we think of black dotted edges are the unique gate into the field.)

$$V = V_T$$

$$E = E_T + (F-1)$$

$$E_T = V-1$$

and so

$$E = V-1 + (F-1) = V+F-2 .$$

Hence

$$\boxed{V - E + F = 2}$$

QED

A soccer ball consists of P pentagonal black fields and H hexagonal white fields.

$$V = \frac{5P + 6H}{3}$$

$$E = \frac{5P + 6H}{2}$$

Now

$$2 = V - E + F$$

$$2 = \frac{5P+6H}{3} - \frac{5P+6H}{2} + P+H$$

$$2 = \frac{10P + 12H - 15P - 18H + 6P + 6H}{6}$$

$$2 = \frac{P}{6}$$

$$\boxed{P = 12}$$

This is true for any sized soccer ball, or fullerene molecule, or dome!