MA310 Solutions : Summer 2010

Q1) $i^{(12)} = .12$

$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1+i \implies i = .126825$

$\left(1+\frac{i^{(3)}}{3}\right)^{3} = 1+i \implies i^{(3)} = .121812$

$\left(1-\frac{d^{(3)}}{3}\right)^{3} = \frac{1}{1+i} \implies d^{(3)} = .117059$

Q2) $\ddot{a}_{\overline{n}|} = 1+v+\cdots+v^{n-1} = \frac{(1-v^n)}{1-v} = \frac{(1-v^n)}{1-\frac{1}{1+i}} = \cdots = \frac{(1-v^n)}{d}$

Q3) $96 \times 1.04^t = 97.39 \implies 1.04^t = \frac{97.39}{96}$

$\implies t = \frac{\ln(97.39/96)}{\ln(1.04)} = $ etc.

Q4) $PV = -2000 - 2000v + 28 \times 12 v^3 a^{(12)}_{\overline{3}|}$

$+ v^5 \Big(28 \times 12 (1.08) v\, a^{(12)}_{\overline{1}|}$

$+ \cdots$

$+ 28 \times 12 (1.08)^{19} v^{19} a^{(12)}_{\overline{1}|} \Big)$

$PV = -2000 - \frac{2000}{1.08} + 28 \times 12 \times \left(\frac{1}{1.08}\right)^3 \left[\frac{1-\left(\frac{1}{1.08}\right)^3}{i^{(12)}} \right]$

$+ v^5 a^{(12)}_{\overline{1}|} \, 28 \times 12 \times \underbrace{a_{\overline{19}|} @ i'}_{=19}$ $\quad \frac{1}{1+i'} = 1.08v = 1$

$\left(1+\frac{i^{(12)}}{12}\right) = 1.08 \implies i^{(12)} = 0.077208$

$PV = €\, 1,028,872$

Q5) $PV = 5\bar{a}_{\overline{3}|} + 2v\,\bar{a}_{\overline{2}|} + \cancel{4}\,2\, v^2 \bar{a}_{\overline{1}|}$ $\quad (*)$

$1.04 = e^{\delta} \qquad \delta = \ln(1.04)$

PTO

$$\bar{a}_{\overline{3}|} = \frac{1-\left(\frac{1}{1.04}\right)^3}{\ln(1.04)} =$$

$$\bar{a}_{\overline{2}|} = \frac{1-\left(\frac{1}{1.04}\right)^2}{\ln(1.04)} =$$

$$\bar{a}_{\overline{1}|} = \frac{1-\left(\frac{1}{1.04}\right)}{\ln(1.04)} =$$

$$v = \frac{1}{1.04} =$$

Now apply (*) to find PV.

Accumulated value $= (1.04)^3 \times PV$

Q6)

a) $F_0 = 5394$ $F_T = 4812$ $t_1 = \frac{3}{12}$ $T = \frac{6}{12}$ $C_{t_1} = 200$

MWRR $= i$ where

$$F_0(1+i)^T + C_{t_1}(1+i)^{T-t_1} = F_T$$

$$5394(1+i)^{\frac{1}{2}} + 200(1+i)^{\frac{1}{4}} - 4812 = 0$$

$$x = (1+i)^{\frac{1}{4}}$$

$$5394x^2 + 200x - 4812 = 0$$

$$x = 0.92615 \Rightarrow i = -26.42\%$$

b) $T = \frac{21}{12}$

$C_{t_1} = 200$ $t_1 = \frac{9}{12}$ $F_{t_1^-} = 4212$

$C_{t_2} = <180>$ $t_2 = \frac{20}{12}$ $F_{t_2^-} = 5400 + 180 = 5580$

$F_0 = 5154$ $F_T = 5400$

TWRR $= i$ where

$$(1+i)^T = \frac{4212}{5154} \times \frac{5580}{4412} \times \frac{5400}{5400} = 1.03407$$

$$1+i = (1.03407)^{12/21} = 1.01933$$

$$i = 1.933\%$$

Q7) Per \$100 shares ③

real value at t=0 dividend @ t=1 $410 \times \frac{147.7}{153.4} = 394.77$

" $t=2$ $460 \times \frac{147.7}{158.6} = 428.39$

" $t=3$ $510 \times \frac{147.7}{165.1} = 456.25$

PV redemption @ t=3 $9300 \times \frac{147.7}{165.1} = 8319.87$

$$-7800 + 394.77v + 428.39v^2 + 8776.17v^3 = 0$$

$$\text{LHS} @ v = \frac{1}{1.05} = 545.719$$

$$\text{LHS} @ v = \frac{1}{1.10} = -483$$

$$545 = m\frac{1}{1.05} + c$$

$$-483 = m\frac{1}{1.1} + c$$

$$-\frac{1025}{0.0453} = m \qquad c = 545 + \frac{1025}{.0453}\frac{1}{1.05}$$

$$= 22\ldots5$$

$0.93 \leq v \leq 0.94$

$v = 0.935 \Rightarrow i = 6.95\ \%$

8) $400\left((1.05)a_{\overline{4}|} + (1.05)^2 v^4 a_{\overline{4}|} + (1.05)^3 v^8 a_{\overline{4}|} + \ldots\right.$

$$= 400\, a_{\overline{4}|}\, 1.05\left(1 + 1.05v^4 + (1.05v^4)^2 + \ldots\right)$$

$$= 400\, a_{\overline{4}|}(1.05)\, \ddot{a}_{\overline{\infty}|}@i'$$

$$\frac{1}{1+i'} = \frac{1.05}{(1.08)^4} = v'$$

$$d' = 1 - v'$$

$$= 400(1.05)\left[\frac{1 - \left(\frac{1}{1.08}\right)^4}{0.08}\right]\frac{1}{d'} = \text{etc.}$$

per £100

Q9)

a) $P = (1-0.2)5a^{(2)}_{\overline{10|}} + 100v^{10}$ at 6%

$a^{(2)}_{\overline{10|}} = \frac{i}{i^{(2)}} a_{\overline{10|}} = \frac{0.06}{0.059126} \frac{1-v^{10}}{0.06}$ $v^{10} = 0.55839$

$P =$ etc.

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b) $P = 5a^{(2)}_{\overline{10|}} + 100v^{10}$

Try two values of i a little above 6%, say $i = 7\%$, $i = 8\%$ and linearly interpolate.

Q10) a) An arbitrage exists if (A): an investor can make a deal that would give an immediate profit with no risk of future loss, or if (B) an investor can make a deal with zero initial costs, no risk of future loss, and a non-zero probability of future profit.

"No arbitrage" implies that any two (combinations of) securities that give exactly the same payments must have the same price.

b) The following have equal cash flows:

A) Enter into a forward contract to buy one unit of S with forward price K maturing at time T, and invest $Ke^{-\delta T}$ at time 0.

B) Buy one unit of S at current price S_0.

The cost of A is $K e^{-\delta T}$ and cost of B is S_0, so

$$K e^{-\delta T} = S_0, \quad K = e^{\delta T} S_0$$

c) $e^{\delta} = (1+i) \Rightarrow e^{\delta T} = (1+i)^T$

$$K = 52.50\, e^{\frac{3}{12}\delta} = 52.50(1.07)^{\frac{3}{12}} = €53.396$$

11) a) $P_4 \times (1+y_4)^4 = 1 \qquad y_4 = \frac{1}{P_4^{0.25}} - 1 = \text{etc}$

$$\cancel{P_4} P_{12} (1+f_{4,8})(\cancel{1+y_4}) = (\cancel{1+y_{12}}) P_4 \cancel{P_{12}}$$

$$P_{12}(1+f_{4,8})^4 = P_4 \qquad f_{4,8} = \left(\frac{P_4}{P_{12}}\right)^{\frac{1}{4}} - 1 = \text{etc.}$$

b) $$F_t = -\frac{1}{P_t}\frac{d}{dt}P_t$$

$$= -\frac{1}{100-2t^2}(-4t)$$

$$= \frac{1}{100-18}(-4 \times 3) = 14.6\%$$