Semester II Examinations 2009/2010

Exam Code(s)	3FM2
Exam(s)	Third University Examination
Module(s)	Actuarial Mathematics I
Module Code(s)	MA310
Paper No	1
Repeat Paper	
External Examiner(s)	Prof. D. Armitage
	Mr P. Cooper
Internal Examiner(s)	Prof. T. Hurley
	Dr. G. Ellis
Instructions:	Attempt **all** questions for an actuarial exemption. Otherwise, attempt the **one** question in Section D and just **two** questions from each of Sections A,B, C and E. Please don't use a red pen.
Duration	**3 hours**
No. of Pages	4 pages (including this page)
Disciplines(s)	Mathematics
Course Co-ordinators(s)	Graham Ellis

Requirements:

MCQ	
Release to Library:	No
Handout	Some useful formulae are printed on page 4
Statistical Tables/ Log Tables	
Cambridge Tables	
Graph paper	
Log Graph Paper	
Other Materials	Calculator which is not capable of storing text

SECTION A (5 marks per question)

1. The nominal rate of interest convertible monthly is 12% per annum. Calculate the nominal rate of interest per annum convertible every 4 months, and the nominal rate of discount per annum convertible every 4 months.

2. Show that

$$\ddot{a}_{\overline{n}|} = \frac{1 - v^n}{d} \ .$$

3. A 182-day government bill, redeemable at €100, was purchased for €96 at the time of issue and was later sold to another investor for €97.89. The rate of return received by the initial purchaser was 4% per annum effective. Calculate the length of time in days for which the initial purchaser held the bill.

SECTION B (10 Marks per question)

4. A project has an initial outlay on 1/5/2010 of €2 000 000. One year later a further expendidture of €2 000 000 will be required. On 1/5/2013 income will be received of €28 000 a month payable in arrears for 22 years. The income increases by 8% per annum compound on 1st May each year starting in 2016.

Calculate the net present value of the project at an interest rate of 8% per annum effective.

5. An individual makes an investment of €5m per annum in the first year, €7m per annum in the second year and €9m per annum in the third year. The investments are made continuously throughout each year. Calculate the accumulated value of the investments at the end of the third year at a rate of interest of 4% per annum effective.

6. The following are total valuations of a pension fund (€000s).

Year	1st Jan	1st April	1st July	1st Oct
2008	5154	5406	5394	4212
2009	4812	4554	4440	5400
2010	6000			

On 30th June 2008 €100 000 worth of equities were sold. On 30th September 2008 equity dividends of €300 000 were received and a company contribution of €200 000 was received on 2nd October 2008. Interest on cash of €60 000 was received on 1st April 2009. Investment managers were paid €180 000 from the fund on 30th September 2009.

(a) Calculate the money weighted rate of return for the last two quarters of 2008.

(b) Calculate the annual effective time weighted rate of return per annum earned on the fund over the period 1st January 2008 to 1st October 2009.

SECTION C (17.5 Marks per question)

7. An investor bought a number of shares at 78 cent each on 31 December 2006. She received dividends on her holding on 31 December 2007, 2008 and 2009. The rate of dividend per share is given in the table below.

Date	*Rate of dividend per share*	*Retail price index*
31 / 12 / 2006	——	147.7
31 / 12 / 2007	4.1 cent	153.4
31 / 12 / 2008	4.6 cent	158.6
31 / 12 / 2009	5.1 cent	165.1

On 31 December 2009 she sold her shares at a price of 93 cent per share.

Calculate, using the retail price index values shown in the table, the effective annual real rate of return achieved by the investor.

8. The current rental on a property is €400,000 per annum and increases by 5% compound at each 4-yearly rent review. The next review is on 1 September 2010. An investor wants to purchase the property on 1 September 2010. What price should she pay to give a yield of 8% per annum effective?

9. A fixed-interest security has just been issued. The security pays half-yearly coupons of 5% per annum in arrear and is redeemable at par 10 years after issue.

(a) Calculate the price to provide an investor with a net redemption yield of 6% per annum effective. The investor pays tax at a rate of 20% on income and is not subject to capital gains tax.

(b) Determine the annual effective gross redemption yield on this security assuming the price calculated in (a) is paid.

SECTION D (15 Marks per question)

10. (a) Describe two types of *arbitrage* and explain the assumption of *no arbitrage* in financial mathematics.

(b) Show that the forward price K for a security with no income is

$$K = S_0 e^{\delta T}$$

where S_0 is the current price of the security, δ is the force of interest, and T is the time when the contract will mature.

(c) A three-month forward contract exists in a zero-coupon corporate bond with a current price per €100 nominal of €52.50. The yield available on three-month government securities is 7% per annum effective. Calculate the forward price.

SECTION E (10 Marks per question)

11. (a) The prices of zero coupon bonds for various terms are as follows:

$$\overline{4|} = €0.95,\ \overline{8|} = €0.80,\ \overline{12|} = €0.45\ .$$

Calculate y_8 and $f_{4,8}$.

(b) The price at time 0 of a zero coupon bond of term $2 < t < 4$ is given by the equation

$$P_t = (100 - 2t^2)\% \text{ nominal.}$$

Calculate the instantaneous forward rate F_3 at time 3.

12. (a) Calculate the 3-year par yield if interest rates are given as follows:

$$y_1 = 5\%,\ Y_2 = 4\%,\ y_3 = 3\%.$$

(b) Calculate the gross redemption yield on a 2-year fixed interest security redeemable at par with annual coupon of 3%, assuming that

$$Y_1 = 3\%,\ Y_2 = 4\%.$$

13. (a) Define *effective duration* and *duration.*

(b) Calculate the convexity of a perpetuity payable annually in advance where the interest rate is 4%.

SOME USEFUL FORMULAE

$$e^{\delta} = 1 + i = \frac{1}{1-d} = (1 + \frac{i^{(m)}}{m})^m = (1 - \frac{d^{(m)}}{m})^{-m}$$

$$a_{\overline{n|}} = \frac{1 - v^n}{i}$$

$$\ddot{a}_{\overline{n|}} = \frac{1 - v^n}{d}$$

$$\overline{a}_{\overline{n|}} = \frac{1 - v^n}{\delta}$$

$$a^{(m)}_{\overline{n|}} = \frac{i}{i^{(m)}} a_{\overline{n|}}$$

$$\ddot{a}^{(m)}_{\overline{n|}} = (1 + i)^{\frac{1}{m}} a^{(m)}_{\overline{n|}}$$

$$\delta = i^{(\infty)}$$