Actuarial Mathematics (MA310)

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Section B Continued: Project Appraisal

Example

An investor borrows money at an effective ROI of 10% pa to invest in a 6-year project with an internal rate of return of 18.7% The cashflows for the project are:

- Initial Outlay of £25,000
- Regular Income £10,000 pa during 1st 5 years (payable continuously)
- Regular expenditure £2,000 pa during 1st 5 years (payable continuously
- Decommissioning expense of £5,000 at end of 6th year.

Calculate the discounted payback period for the project.

DPP=t where

$$A(t) = <25,000> (1+i)^t + (10000 - 2000)\bar{s}_{\overline{t}|} = 0$$

$$\Rightarrow \bar{a}_{\overline{t}|} = \frac{1 - v^t}{\delta} = \frac{25,000}{8000}$$

$$t = 3 \text{ yrs } \ 8\frac{1}{2} \text{ mnths}$$

Example

	Initial Outlay	Proceeds
Project C	£100,000	£140,000 at end $\overline{5}\vert$
Project D	£100,000	£38,850 pa in arrears for $\overline{3}\vert$

For both projects calculate:

1. IRR
2. Range of ROI at which money can be borrowed for the project to be viable
3. Accummulated profit @ end $\overline{5}\vert$ assuming projects are financed by a loan subject to interest at 6.25%

What other considerations might be taken into account when deciding between Project C and Project D?

Internal Rate of Return Proj C:

$100,000(1 + IRR_C)^5 = 140,000 \Rightarrow IRR_C = 7.0\%$

Proj D: $38850 a_{\overline{3}|@IRR} = 100,000 \Rightarrow IRR_D = 8.1\%$

Range for ROI

Borrowing ROI $<$ IRR $\Rightarrow$ Profit

$< 7.0\% \Rightarrow$ C & D profitable

$> 7.0\%$ & $< 8.1\ \% \Rightarrow$ D profitable, C not profitable

$> 8.1\% \Rightarrow$ C & D not profitable

Accummulated profit

Proj C: $140,000 - 100,000(1.0625)^5 = 4592$

Proj D: $38850 s_{\overline{3}|} \times (1.0625)^2 - 100,000 \times (1.0625)^5 = 4561$

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- Can the investors fund loan repayments for project C since it does not produce any proceeds for $\overline{5}|$?

Measures of investment return

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- pension funds, investment company funds, mutual funds
- measure actual return acheived
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- Investment return:
  - Investment income
  - Capital gains/losses
- Non-investment return
  - New money/withdrawals

Example

The market value of a small pension fund's assets was £2.7m @ 1/1/99, £3.0m @30/4/99 and £3.1m @ 31/12/99 . During 1999 the only cash flows were:

bank interest and dividends	£125,000	@ 30/6/99
proceeds of sale of shares	£100,000	@ 1/8/99
lump sum retirement benefit	£75,000	1/5/99
company contribution	£50,000	@31/12/99

Calculate the investment return for 1999.

Method 1:

$$27000(1+i)+<75>(1+i)^{\frac{8}{12}}+50=3,100$$

So $i = 16.0\%$

Why are the £100,000 and £125,000 not included?

Method 2:

$$\frac{3000}{2700} \times \frac{3100 - 50}{3000 - 75} = 1.159$$

So $i = 15.9\%$

Which method is the correct one?

Money Weighted Rate of Return (Method 1 above)
MWRR is the effective ROI i that solves

$$F_0(1+i)^T + \sum_{j=1}^{n} C_{t_j}(1+i)^{T-t_j} = F_T$$

where
F_0 = fund size at time 0
F_T = fund size at time T
C_{t_j} = non-investment return net cashflow at time t_j

i is influenced by the size and timing of cashflow C_{t_j}
MWRR gives the actual RR achieved by the fund.

Time Weighted Rate of Return (Method 2 above)

TWRR is the effective ROI i per annum that solve

$$(1+i)^T = \frac{F_{t_1^-}}{F_0 + C_0} \times \frac{F_{t_2^-}}{F_{t_1^-} + C_{t_1}} \times \frac{F_{t_3^-}}{F_{t_2^-} + C_{t_2}} \times \cdots \times \frac{F_T}{F_{t_n^-} + C_{t_n}}$$

where $F_{t_j^-}$ is the value of the fund *immediately before* cashflow C_{t_j}.

Note: i is independent of the size and timing of cashflows.

TWRR gives a RR more suitable for assessing the performance of the Investment Manager (who has no control over the size or timing of the non-investment CFs).

Example

Consider a lottery winner's investment protfolio.

	Fund Value	Caqshflow	Details
1 Jan	75,000		
31 March	90,000		
1 April	2,700,000	win jackpot	
30 Sept	2,600,000		
1 October		<2,500,000>	withdrawal
31 Dec	125,000		

MWRR

$$125 = 75(1+i) + 2700(1+i)^{\frac{9}{12}} + < 2500 > (1+i)^{\frac{3}{12}}$$

Try $i = 0\%$ to get $F_T = 275$. So need $i < 0\%$.

Try $i =< 20 > \%$ to get $F_T =< 20 >$. So need $i >< 20 > \%$.

Linear interpolation gives $i =< 10.2 > \%$.

TWRR

$$(1+i)^1 = \frac{90}{75} \times \frac{2600}{2700+90} \times \frac{125}{2600+2500} = 1.39\%$$

So $i = 40\%$

MWRR puts almost all the weight on the middle period when the fund was the largest and returns were negative.

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- TWRR also requires fund values at all the CF dates
- MWRR may not have a unique solution (or indeed any solution)

Linked Internal Rate of Return

LIRR - a compromise

$$(1+i)^{t_n} = (1+i_1)^{t_1} \times (1+i_2)^{t_2 . t_1} \times (1+i_3)^{t_3 - t_2} \times \cdots \times (1+i_n)^{t_n - t_{n-1}}$$

where i_k = effective ROI (annual ROI) earned in interval $[t_{k-1}, t_k]$.

If the time intervals $[t_{k-1}, t_k]$ are chosen so that CFs occur at times t_k then LIRR=TWRR.

The LIRR intervals are arbitrary (eg. monthly)

In practice the $(1+i_k)^{t_k - t_{k-1}}$ are calculated using MWRR (for the month or quater) or by a more appropriate method.

Example

A life office operates a Far East fund which achieved quaterly MWRR of 4.1%, 2.8%, 1.7% and 2.1% during 1999.

Calculate the LIRR.

$$(1+i)^1 = ((1.041)^4)^{\frac{1}{4}} \times 1.028 \times 1.017\ 1.021 = 1.111$$

LIRR $= 11.1\%$

The following are total valuations of a pension fund (£000s).

Year	1st Jan	1st April	1st July
2004	1859	1901	1999
2005	1902	1759	2043
2006	2100		

On 30th June 2004 £40,000 worth of equities were sold. On 31st March 2004 equity dividends of £15,000 were received together with a company contribution of £120,000. Investment managers were paid £20,000 from the fund and interest on cash of £30,000 was received on 2nd April 2005.

1. Calculate the annual effective time weighted rate of return per annum earned on the fund over the period 1st January 2004 to 1st July 2005.
2. Calculate the annual effective linked internal rate of return on the fund over the same period as (1) by linking returns over half year periods. The half year returns that are to be linked are money weighted rates of return.

Non-investment cashflows were: 31st March 04 (120K) and 2nd April 05 ($< 20 >$).

(a)

$$(1+i)^{\frac{3}{2}} = \frac{1901-120}{1859} \times \frac{1759}{1901} \times \frac{2040}{1759-20}$$

So $i = 2.64\%$ per annum

(b)
1/1/04 - 30/6/04

$$1859(1+i)+120(1+i)^{\frac{1}{2}}=1999$$

Let $X=(1+i)^{\frac{1}{2}}$ and solve

$$1859X^2+120X-1999=0$$

to find $X=1.005198$ and 6 monthly return $=.0104229$.

1/7/04 - 31/12/04
6 monthly return $=\frac{1902}{1999}-1=-.048524$

31/12/04 -30/06/05

$$1902(1+i)-20(1+i)^{\frac{1}{2}}=2040$$

and 6 monthly return $=.0835$.

LIRR:

$$(1+i)^{\frac{3}{2}} = (1.0104229)(1 - .048524)(1.0835)$$

and the LIRR is $i = 2.759\%$ per annum

Another example

The following data relates to the assets of an investment fund.

Date	Market value
1/1/97	£4.2m
1/1/98	£4.6m
1/1/99	£5.4m
1/7/99	£5.1m
31/12/99	£5.5m

The only cashflow during the calendar years 97-99 that was not generated by the assets of the fund was a payment of £200,000 received by the fund on 30 June 1999.

For the period 1/1/97 - 31/12/99 calculate: (a) MWRR, (b) TWRR, (c) LIRR. (Latter to be calculated using year long linking periods.

MWRR

$$4.2(1+i)^3 + 0.2(1+i)^{\frac{1}{2}} = 5.5$$

and $i = 8.0\%$

TWRR

$$(1+i)^3 = \frac{5.1-0.2}{4.2} \times \frac{5.5}{5.1} = 1.2583$$

and $i = 8.0\%$

LIRR

1997:

$$4.2(1 + i_1) = 4.6 \Rightarrow 1 + i_1 = 1.0952$$

1998:

$$4.6(1 + i_2) = 5.1 \Rightarrow 1 + i_2 = 1.1087$$

1999:

$$5.1(1 + i_3) + 0.2(1 + i_3)^{\frac{1}{2}} = 5.5 \Rightarrow 1 + i_3 = 1.0385$$

$$(1 + i)^3 = 1.0952 \times 1.1087 \times 1.0385 = 1.261$$

and LIRR $= 8.0\%$ per annum