

# Actuarial Mathematics (MA310)

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## Section C: Investments contd.

# Convertibles

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### Income

- ▶ Fixed annual interest payment/dividend.
- ▶ Generally higher than ordinary shares & lower than conventional loan stocks/ preference shares.

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Investors may choose not to convert into ordinary shares and redeem at par instead (or hold for later redemption).



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- ▶ As the conversion date approaches it becomes clearer whether it will stay as a loan stock/preference share or become ordinary shares & this will be reflected in volatility of return.
- ▶ Less volatility than in the price of the underlying equity.

### **Attraction for an investor:**

Opportunity to combine lower risk of a debt security with potential for large gains of an equity. The price paid is a lower income yield than on a normal loan stock or preference share. The option to convert will have a time value which will be reflected in the price of the stock.

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- ▶ Maintenance expenses are paid out of rental income. The tenant is often responsible for building maintenance and insurance costs.

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- ▶ Buying and selling expenses are much higher than for shares and bonds.
- ▶ Each property is unique and can be difficult to value. Valuation is expensive because of the need to employ an experienced surveyor.

**Security:**

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- ▶ Neither rental income or capital value are guaranteed.
- ▶ Even poor performing properties will have value  $\geq 0$ .
- ▶ Property markets can crash just like shares.

## Marketability:

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- ▶ Marketability is poor.
- ▶ Dealing costs are high.
- ▶ Large unit size. You can't easily sell part of a property as can be done with shares and bonds.
- ▶ Value obtainable on a sale is uncertain (compared with sale of equities).

**Return:**



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- ▶ Rental yield generally lower than government bonds because of expectation of capital growth.
- ▶ Capital values can be volatile in the short term.

## Example

A property pays rent annually in arrears and has rent reviews every  $n$  years. The most recent review has just been completed and the rent for the next  $n$  years will be  $R$  per annum. It is expected that rents will increase by a rate of  $g$  compound at future rent reviews in perpetuity. Find the price required to yield  $i$  per annum effective on the property.

Rent  $R$  p.a. next  $n$  years.

Rents grows  $g$  every  $n$  years.

$P$  = price.

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$$P = Ra_{\overline{n}|} + (1 + g)v^n Ra_{\overline{n}|} + (1 + g)^2 v^{2n} Ra_{\overline{n}|} + \dots$$



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$$\begin{aligned} P &= Ra_{\overline{n}|} + (1 + g)v^n Ra_{\overline{n}|} + (1 + g)^2 v^{2n} Ra_{\overline{n}|} + \dots \\ &= Ra_{\overline{n}|} \left( 1 + \frac{1 + g}{(1 + i)^n} + \left( \frac{1 + g}{(1 + i)^n} \right)^2 + \dots \right) \end{aligned}$$

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$$= Ra_{\overline{n}|} \left( 1 + \frac{1 + g}{(1 + i)^n} + \left( \frac{1 + g}{(1 + i)^n} \right)^2 + \dots \right)$$

$$= Ra_{\overline{n}|} i \ddot{a}_{\infty|i'}$$

$$\text{where } \frac{1}{1 + i'} = \frac{1 + g}{(1 + i)^n}$$

$$= Ra_{\bar{n}|i}/d_{i'} = Ra_{\bar{n}|i} \frac{1+i'}{i'}$$

## Example

An investor who is taxed at 30% purchased 1000 shares on 30th April, 2006, in a company ex dividend. All income tax on income received in a calendar year is paid on 31st March of the following year. Dividends are paid annually and the next dividend of £2.50 per share will be paid 5 months after the date of purchase. The dividends are expected to rise by 4% each calendar year. Calculate the price paid by the investor if the expected net yield is 10% per annum effective.

Price =  $P$

$i = 0.1$

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$$P = 1000(2.5(1.04)v^{1\frac{5}{12}} + 2.5(1.04)^2v^{2\frac{5}{12}} + \dots)$$

$$- (.3)1000(2.5(1.04)v^{1\frac{11}{12}} + 2.5(1.04)^2v^{2\frac{11}{12}} + \dots)$$

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$$P = (1000)(1 - .3\sqrt{v})(2.5)v^{\frac{5}{12}}(1.04v + (1.04)^2v^2 + \dots)$$

$$P = 2500(1 - .3\sqrt{v})v^{\frac{5}{12}} a_{\infty|i'}$$

where

$$\frac{1}{1+i'} = \frac{1.04}{1.1} \Rightarrow i' = \frac{.06}{1.04}$$



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$$P = 2500(1 - .3\sqrt{v})v^{\frac{5}{12}} \frac{1.04}{.06} = 29,734$$

## Example

A four-year index-linked security was issued on 1st April 2002. It pays nominal coupons of 8% every two years in arrear and is redeemed at 110%. The coupons and capital payment are inflated by reference to the inflation index 6 months before the payment is made. The inflation index on 1st October 2001 was 601 and the table below shows the index values at other times.

1/4/02	1/10/03	1/4/04	1/4/05	1/10/05	1/4/06
631	662	695	730	788	877

- (a) Calculate the effective rate of interest per annum (assuming that it is constant) earned over the life of the index linked security if the price of the stock is £100 per £100 nominal on 1st April 2002.
- (b) Calculate the real rate of return per annum over the life of the index linked security.

(a) Per 100 nominal:

$$100 = 8v^2 \frac{Q(1/4/04 - .5)}{Q(1/4/02 - .5)} + 118v^4 \frac{Q(1/4/06 - .5)}{Q(1/4/02 - .5)}$$

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$$154.72v^4 + 8.81v^2 - 100 = 0$$

Formula for roots of quadratic (ignoring negative root) yields:

$$v^2 = .775979$$

$$i = 13.52\%$$

(b) Per 100 nominal:

$$100 = 8.81v'^2 \frac{Q(1/4/02)}{Q(1/4/04)} + 154.72v'^4 \frac{Q(1/4/02)}{Q(1/4/06)}$$



(b) Per 100 nominal:

$$100 = 8.81v'^2 \frac{Q(1/4/02)}{Q(1/4/04)} + 154.72v'^4 \frac{Q(1/4/02)}{Q(1/4/06)}$$

Then, as in part (a),

$$v'^2 = .912541$$

$$i' = 4.68\%$$

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Give an outline how corporate debt differs from government debt under the headings: security, marketability, yield etc. .

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**Security:** Government bonds generally zero default risk for income & capital. Company debt less secure. Company debt level depends on type of bond. Debentures which are secured on assets of the company have better security than unsecured loans.

**Marketability:** Government bonds usually more marketable corporate debt because they are issued in larger volumes. Fewer investors hold corporate debt and trade is less active. Marketability poorest on small unlisted corporate issues.

**Yield:** Lower for government bonds than equivalent corporate debt because of greater security and marketability.

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**Other:** Government bonds have lower dealing costs.