

Actuarial Mathematics (MA310)

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Section C: Investments

Fixed interest government borrowings

Fixed interest borrowings by other bodies

Shares and other equity type borrowings

Property

Derivatives

Fixed Interest Government Borrowings

Fixed Interest Government bonds:

Government body borrows by floating a loan on a stock exchange.

Terms of issue are set by borrower.

Investors are invited to subscribe to loan:

- (i) at a given fixed issue price
- (ii) or investor tenders a price & loan issued to highest bidders (subject to minimum tender prices).

Allocation rules apply where the issue is oversubscribed.

Example

Terms of issue:

Issue date	1 March 2001
Nominal Amount	£2,000,000,000
Coupon rate	4.25% pa payable half yearly
Redemption date	1 March 2018
Redemption price	£100 per £100 nominal (This is 'redemption at par')
Issue price	£98.50 per £100 nominal

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- ▶ The investor might sell the bonds on stock exchange, or buy them on stock exchange after issue date.

Government bonds: Redemption date

Redemption date is sometimes a range (eg 1/3/16 -1/3/19) during which the borrower may choose to redeem at an interest payment date.

Sometimes no redemption date - known as irredeemable

Government bonds: Strips

Sometimes the interest and redemption proceeds can be separated into two parts

1. a zero coupon bond and
2. a bond redeemable at zero

This is relevant when investor wants to trade in the bond after it is issued.

Government bonds: Tax

Each investor is subject to the taxation regime particular to her/it.

eg: Taxation at rate $t_1\%$ on interest payments

Taxation at rate $t_2\%$ on capital gains

Investors taxation regime versus other investors' will influence the attractiveness of bonds.

Government bonds: Security

Generally very low default risk. (Depends on the government/state)

Government bonds: Marketability/Liquidity

Generally investors can deal in large quantities with little or no impact on the market price.

Low dealing costs

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- ▶ Low volatility usually means low expected return relative to higher volatility investments.
- ▶ Income and redemption cashflows are fixed in money terms but not in real terms.

Index linked bonds

Interest and redemption payments are linked to an inflation index.

There is a small time lag between date of index value and publication of the index value. Administration of interest payments requires monetary amounts to be known shortly in advance. So there is no inflation protection during this short lag period.

Cashflow model (assuming previous example is index linked and ignoring the time lag): the half-yearly interest payment at time t will be $£23.63 \times \frac{\text{Inflation index @}t}{\text{Inflation index @}0}$

Example

Consider a 3.5% coupon stock issued in February 1995 and redeemed in February 2000. The coupon payments are made each year and are linked to an inflation index with a one-year time lag. The index values each February, from 1994 to 2000 are :

Year	1994	1995	1996	1997	1998	1999	2000
Index	540	562	584	607	632	657	788
Inflation		4.07	3.91	3.94	4.12	3.96	19.94

The base month for indexation is February 1994. Because of the time lag:

Year	1996	1997	1998	1999	2000
Coupon payments per £100	3.64	3.79	3.93	4.10	4.26
Real values in Feb 95 terms	3.50	3.51	3.49	3.51	4.26

With no time lag the real value would be 3.5 in each case. (Note:
 $3.93 = 3.5 \times 607/540$ and $3.49 = 3.93 \times 562/632$)

Example

A fixed interest stock pays a coupon of 8% per annum quarterly, the next coupon is due on 31 May 2006. It is redeemable at 115% at the option of the borrower on any 1st March between 2025 and 2029 inclusive.

Calculate the price an investor would pay on 1st June 2006 to receive an effective net yield of at least 10% per annum. The investor pays tax at 30% on 31st August on income received during the previous calendar year and capital gains tax at 40% on 31st December following realization of any gain.

$D = .08$ next due 31/5/06

$p = 4$

$R = 1.15$

$A = ?$ 1/6/06 (amount investor is willing to pay)

$i \geq .1$

$t_1 \geq .3$ 31/8 on income per calendar year

$t_2 = .4$ 31/12 after gain

$$(1 - t_1) \frac{D}{R} = .048696$$

$$\left(1 + \frac{i^{(4)}}{4}\right)^4 = 1.1 \Rightarrow i^{(4)} = .096444$$

Since $i^{(4)} > .048696$ assume the worst case: i.e. that the borrower redeems at latest possible date.

There is a capital gain at redemption. Per 100 nominal *untaxed* profit would be

$$8a_{\overline{22.75}|}^{(4)} + 115v^{22.75} - A$$

$$PV(\text{income tax}) = .3(4v^{1.25} + 8v^{2.25} + \dots + 8v^{23.25} + 2v^{24.25})$$

$$= (4v^{1.25} + 8v^{1.25} a_{\overline{22}|} + 2v^{24.25})(.3)$$

$$\text{Cgt} = .4(115 - A)v^{23.25}$$

$$\text{So } A = 8a_{\overline{22.75}|}^{(4)} + 115v^{22.75}$$

$$- .3(4v^{1.25} + 8v^{2.25} + \dots + 8v^{23.25} + 2v^{24.25})$$

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Using $v = 1/1.1$ we get $A = 64.67$ per 100 nominal.

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- ▶ Sometimes the rate of interest is annualized.
- ▶ Security: almost zero default risk.
- ▶ Marketability: can be sold quickly at low cost and without affecting market price.

contd.

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- ▶ Often used as benchmark risk-free short-term investment.

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$$\text{par value} \times \left(1 - 0.1 \times \frac{182}{365}\right) = 10,000 \text{ so par value} = \text{£}10,525.$$

Fixed interest borrowings by other bodies

Corporate debt - long term borrowings

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- ▶ Marketability: less marketable than government bonds (mainly because the size of issues are much smaller)
- ▶ Return: higher return than similar government bonds to compensate for lower security and marketability

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 - ▶ type of bond (secured/unsecured)
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- ▶ Types of bond:
 - ▶ **Debenture** (secured)
Loan is secured on some or all of the assets of the company.
If company fails to pay interest or capital then bond holders may take possession of those assets.
 - ▶ **Unsecured loan stocks**
loan not secured on any particular assets
if company fails to pay interest or capital then bondholders are like any other creditors of the company (eg. suppliers). They may apply to the courts to have the company wound up.
Lower security than debentures

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- ▶ Marketability: better than company loan stocks; in some cases as good as government bonds; issues can be very large.

- ▶ Returns: depends on issuer and issue size; higher than yield on similar debenture; lower than yield on similar company unsecured loan.

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- ▶ eg. Investor may select the currency in which interest and/or capital is paid.

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- ▶ Active secondary market.

Example

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- ▶ The security of the bond (based on the quality of the company)
- ▶ Marketability (based largely on size of issue)

Shares and other equity type borrowing

- (A) Ordinary shares
- (B) Preference shares
- (C) Convertibles

Ordinary shares (equities)

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- ▶ Shareholders own the company and have voting rights.
- ▶ Redemption:
 - ▶ Equities do not have a redemption rate.
 - ▶ Investors 'redeem' their capital by selling their shares to someone else (Stock Exchange)
 - ▶ Shareholders make a profit (or loss) on the shares equal to the difference between the purchase amount and the sale amount. This capital gain/loss is an important element of the investment return.

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- ▶ The dividend may be higher/lower/same as last year. It could be zero.

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- ▶ Market sentiment!

▶ Investor's cashflow model (after sale).

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.3	100	Dividend
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1.3	105	
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- ▶ After the date of purchase the future cashflows are unknown (so assumptions are required to construct cashflow model).

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- ▶ Shareholders may lose a significant part of sums invested.

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- ▶ Usually better than loan capital from same company.
- ▶ Not as good as government bonds.
- ▶ investors trade shares more frequently than loan capital.
- ▶ Dealing costs & market spreads higher on smaller stocks.

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- ▶ Unpaid preference dividends are carried forward and may be paid at a later date.

Redemption: Sometimes there is a redemption date.

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Security:

- ▶ Income: greater than ordinary and less than capital.
- ▶ Capital: greater than ordinary and less than loan capital.

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Return: Volatility a little greater than for loan capital and much less than for ordinary shares.