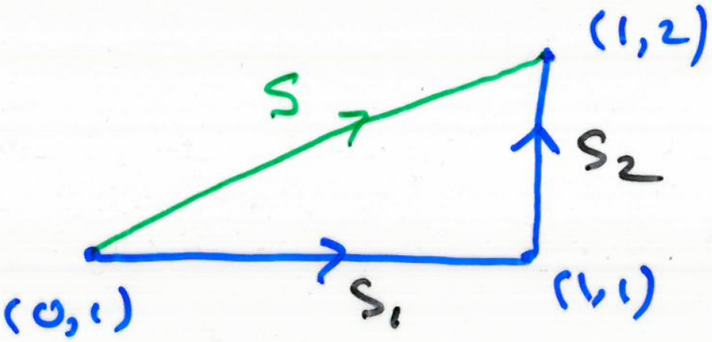
Example Evaluate

$$L' = \int_{S'} (x^2 - y)\,dx + (y^2 - x)\,dy$$

where S' is the line from $(0,1)$ to $(1,1)$ followed by the line from $(1,1)$ to $(1,2)$.

Sol^n



$$L' = \int_{S_1} (x^2-y)\,dx + (y^2-x)\,dy + \int_{S_2} (x^2-y)\,dx + (y^2-x)\,dy$$

The line $y=1$ contains S_1

" " $x=1$ " S_2

$$= \lim_{\|P\| \to 0} \sum_{i=1}^{n} (x_i^2 - 1)(x_i - x_{i-1}) + (1^2 - x_i).0$$

$$+ \lim_{\|P\| \to 0} \sum_{i=1}^{n} (1^2 - y_i) \cdot 0 + (y_i^2 - 1)(y_i - y_{i-1})$$

$$= \int_0^1 (x^2 - 1)\, dx + \int_1^2 (y^2 - 1)\, dy$$

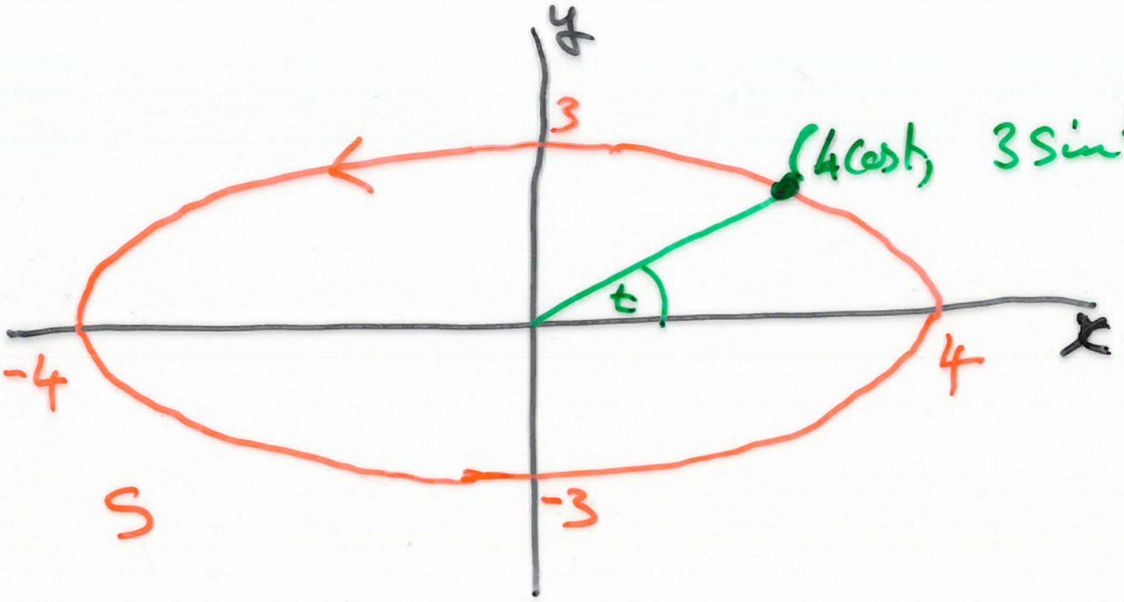
$$= \dots$$

$$= \frac{8}{3}.$$

<u>Example</u> Work is represented by the 1-form

$$\omega = (3x - 4y + 2z)\, dx + (4x + 2y - 3z^2)\, dy + (2xz - 4y^2 + z^3)\, dz$$

Find the work done in moving a particle once around the following ellipse in the xy-plane, in the anti-clockwise direction.



<u>Sol</u>

Soln

$$\text{work} = \int_S (3x - 4y)\,dx + (4x + 2y)\,dy$$

$$x = 4\cos t$$
$$y = 3\sin t$$

$$dx = -4\sin t\,dt$$
$$dy = 3\cos t\,dt$$

work =

$$\int_{t=0}^{2\pi} \Big(3(4\cos t) - 4(3\sin t)\Big)(-4\sin t)\,dt + \Big(4(4\cos t) + 2(3\sin t)\Big)\,3\cos t\,dt$$

$$= \cdots$$

$$= \int_0^{2\pi} 48 - 30(\sin t)(\cos t)\,dt$$

$$= 48t - 15 \sin^2 t \Big|_0^{2\pi}$$

$$= 96\pi$$

Partial Derivatives

Give a 0-form

$$w = f(x, y, z)$$

we denote by

$$\frac{\partial f}{\partial x}$$

the 0-form obtained by regarding y and z as constants and differentiating with respect to x.

We call $\frac{\partial f}{\partial x}$ the partial derivative of f with respect to x.

Example Consider the function

$$f(x,y,z) = \sqrt{1-(x^2+y^2+z^2)}$$

defined on

$$S = \{(x,y,z) \in \mathbb{R}^2 : x^2+y^2+z^2 \leq 1\}.$$

Calculate $\dfrac{\partial f}{\partial x}$.

Sol^n

$$f(x,y,z) = \left(1-(x^2+y^2+z^2)\right)^{\frac{1}{2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}\left(1-(x^2+y^2+z^2)\right)^{-\frac{1}{2}}(-2x)$$

$$= \frac{-x}{\sqrt{1-(x^2+y^2+z^2)}}$$

Similarly:

$$\frac{\partial f}{\partial y} = \frac{-y}{\sqrt{1-(x^2+y^2+z^2)}}$$

$$\frac{\partial f}{\partial z} = \frac{-z}{\sqrt{1-(x^2+y^2+z^2)}}$$

NOTATION

We often write

$$f_x$$

in place of

$$\frac{\partial f}{\partial x}$$