__Example__ find the 1-form

$$\omega = A\,dx + B\,dy + C\,dz$$

describing the function "work" in the constant force field where displacement of a particle from

$(0,0,0)$ to $(4,0,0)$ needs 3 units of work

$(1,-1,0)$ to $(1,1,0)$ " 2 " "

$(0,0,0)$ to $(3,0,2)$ " 5 " " .

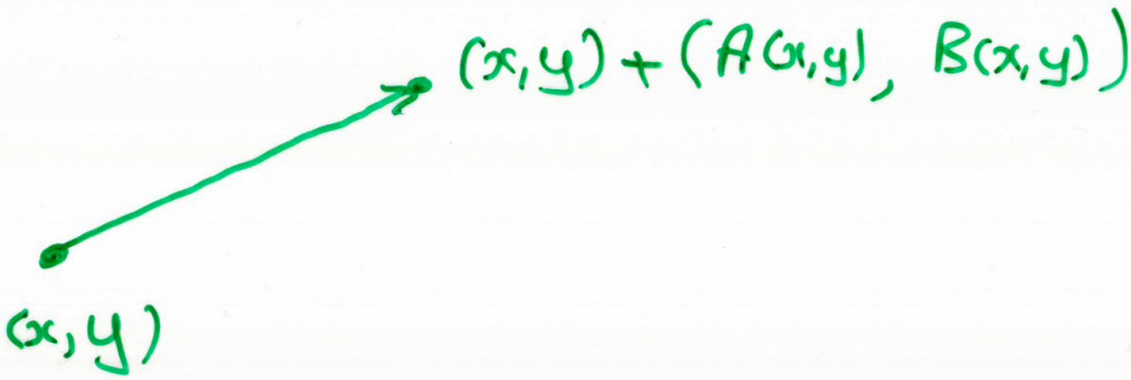
__Sol^n__

$$\left.\begin{aligned} 4 \cdot A &= 3 \\ 2B &= 2 \\ 3A + 2C &= 5 \end{aligned}\right\} \qquad \begin{aligned} A &= \frac{3}{4} \\ B &= 1 \\ C &= \frac{11}{8} \end{aligned}$$

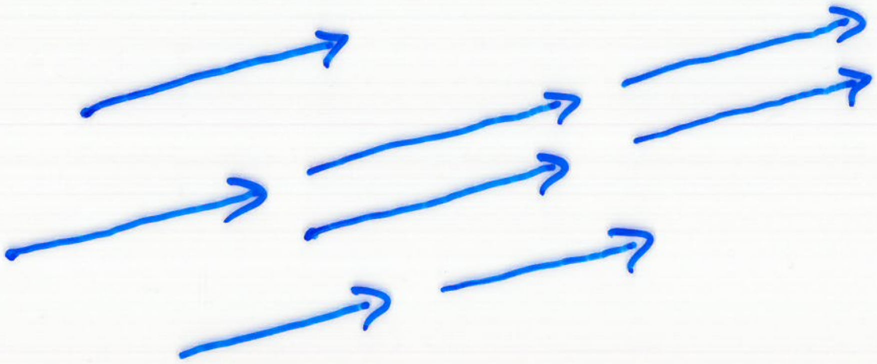
We can think of a 1-form

$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

as a collection of arrows in the plane. For each point (x,y) in the plane we have the arrow



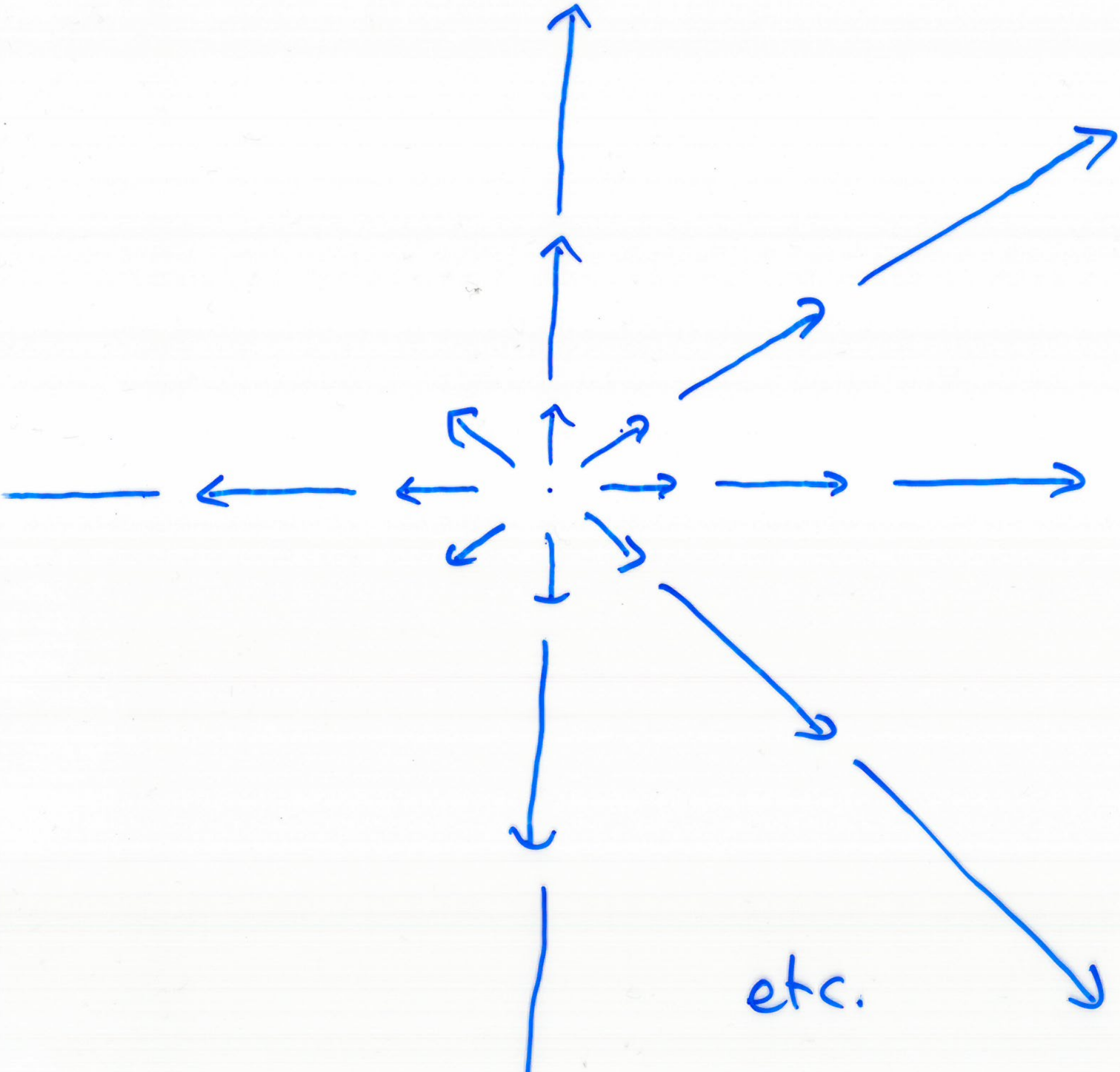
<u>Example</u> The 1-form $\omega = 2dx + dy$ can be pictured as



Example The 1-form

$$\omega = x\,dx + y\,dy$$

can be pictured as



<u>Integration of 1-forms</u>

Let $\omega = A(x,y)\,dx + B(x,y)\,dy$ be a 1-form.

Let $S \subseteq \mathbb{R}^2$ be a 1-dimensional, oriented, connected subset.

<u>Informally</u> if we think of ω as a "work 1-form" then

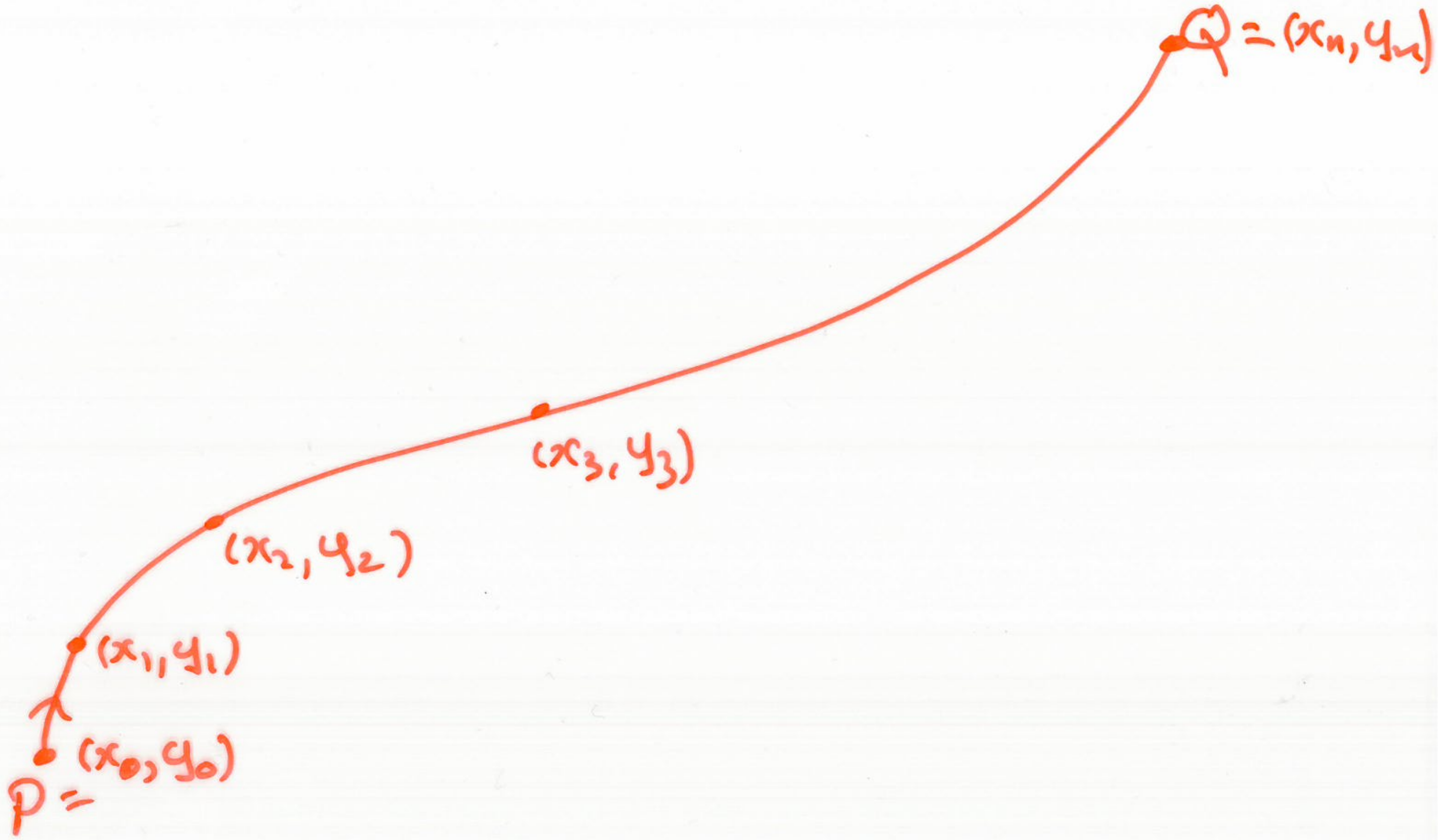
$$\int_S A(x,y)\,dx + B(x,y)\,dy$$

is the total work done in moving a particle from P to Q.

More Formally

$$\int_S A(x,y)\,dx + B(x,y)\,dy =$$

$$\lim_{\|P\|\to 0} \sum_{i=1}^{n} A(x_i^*, y_i^*) \cdot (x_i - x_{i-1}) + B(x_i^*, y_i^*) \cdot (y_i - y_{i-1})$$



where :

$P = \{ (x_0, y_0), (x_1, y_1), \cdots, (x_n, y_n) \}$ is a sequence of points on S, with (x_0, y_0) the initial point of S, and (x_n, y_n) the final point.

$$\|P\| = \max_{1 \le i \le n} \| (x_i, y_i) - (x_{i-1}, y_{i-1}) \|$$

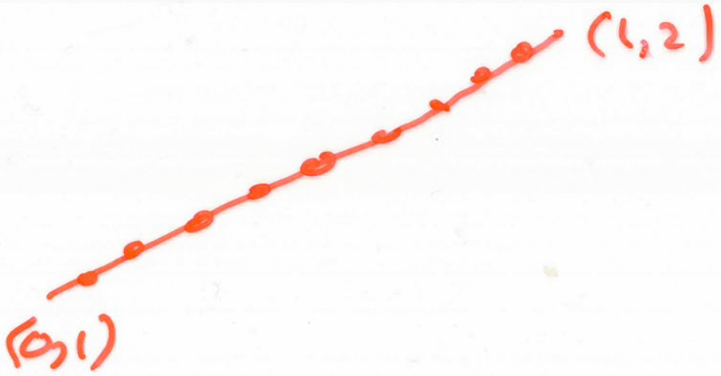
with $\|(x,y)\| = \sqrt{x^2 + y^2}$

<u>Example</u> Let S be the line segment from $(0,1)$ to $(1,2)$.

Evaluate

$$L = \int_S (x^2 - y)\, dx + (y^2 + x)\, dy$$

$\underline{\text{Sol}^n}$ The line $y = x + 1$ passes through $(0,1)$ and $(1,2)$



$$P = \{(x_0, y_0), (x_1, y_1), \cdots, (x_n, y_n)\}$$

$$= \{(x_0, x_0+1), (x_1, x_1+1), \cdots, (x_n, x_n+1)\}$$

$$L = \lim_{\|P\| \to 0} \sum_{i=1}^{n} (x_i^2 - x_i - 1)(x_i - x_{i-1}) + \left[(x_i+1)^2 + x_i\right](x_i - x_{i-1})$$

$$= \int_0^1 (x^2 - x - 1) + \left[(x+1)^2 + x\right] \, dx$$

$$= \int_0^1 2x^2 + 2x \, dx$$

$\vdots$

$$= 5/3$$