A function $\phi = \phi(x, y, z)$ is harmonic if it is continuous and its average over any ball in its domain of definition equals to its value at the centre of the ball:

$$\phi(\xi) = \frac{3}{4\pi r^3} \int_D \phi(x, y, z) \, dx \, dy \, dz$$

Theorem: $\phi$ is harmonic if and only if

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$ 

Electrostatics concerns a change density $\rho = \rho(x, y, z)$ and a potential $\phi = \phi(x, y, z)$. It is postulated that:
\( \varepsilon \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) + \rho = 0 \)  
\( \text{++} \)  

i.e. the potential \( \phi \) harmonic at places where there is no charge.

Equation (\( \text{+++} \)) is called \textit{Poisson's equation},

\( \varepsilon \) is called the \textit{dielectric constant}.

We can consider the \textit{1-form}

\[ E = -d\phi = E_1 \, dx + E_2 \, dy + E_3 \, dz \]

which is called the \textit{electric force field}. It describes the work needed to displace a charge.

The \textit{2-form}

\[ D = -\varepsilon \left( \frac{\partial \phi}{\partial x} \, dy \wedge dz + \frac{\partial \phi}{\partial y} \, dz \wedge dx + \frac{\partial \phi}{\partial z} \, dx \wedge dy \right) \]

is called the \textit{electric displacement}.
Poisson's equation (***) can be expressed as
\[ \rho \, dx \, dy \, dz = dD \] (***)

Electrodynamics

A moving particle is acted on by forces other than just those of electrostatics — namely magnet force.

Faraday postulated (in our language!) that magnetic forces are described by a 2-form

\[ B = B_1 \, dx \, dy + B_2 \, dy \, dz + B_3 \, dz \, dx \]

and that the relationship to electric forces is governed by
\[ d(E \, dt + B) = 0. \]

Here

\[ E \, dt = E_1 dx \, dt + E_2 dy \, dt + E_3 dz \, dt \]

The 2-form \( E \, dt + B \) is called the **electromagnetic field**. The charge and its motion are described by a 3-form

\[ J = \rho \, dx \, dy \, dz + j_1 \, dy \, dz \, dt + j_2 \, dz \, dx \, dt - j_3 \, dx \, dy \, dt \]

called the **moving charge** (or current)
Maxwell's Equations

Electromagnetism is a mathematical theory based on the following definitions:

1) Electromagnetic Field = $E \cdot dS + B$ (2-term)

2) Current = $J$ (3-term)

3) $D = E_1 dS + E_2 dS_1 dx + E_3 dx dy$ (2-term)

4) $H = \frac{1}{\mu} B_1 dx + \frac{1}{\mu} B_2 dy + \frac{1}{\mu} B_3 dz$ (1-term)

$\mu$ = magnetic permeability.
The theory is described by the following equations:

- \( d (E \cdot dt + B) = 0 \)  
  Faraday's Law

- \( d J = 0 \)  
  Gauss's Law

- \( d (D - H \cdot dt) = J \)  
  Ampère/Maxwell's Law