Differential 0-forms in 1 variable

A differential 0-form in 1 variable is just a differentiable, real valued function

$$\omega = f(x)$$

Examples

$$\omega = 3x-4$$
$$\omega = 3x^2+4$$
$$\omega = \sin(x)$$

are all differential 0-forms.

Usually a differential 0-form is given in the context of some closed interval

$$S = [a, b] \subseteq \mathbb{R}$$

or a union of closed intervals

$$S = [a_1, b_1] \cup [a_2, b_2] \cup \dots \cup [a_k, b_k]$$

we only require ω to be differentiable at points in S.

Example

$$\omega = |x|$$

is a differential 0-form on

$$S = [1, 100].$$

clearly ω is <u>not</u> a differential 0-form on $S = [-1, 100]$.

Terminology

will say <u>0-form</u> instead of differential 0-form.

For $a < b \in \mathbb{R}$ we write

$$[a, b] = \{ x \in \mathbb{R} : a \leq x \leq b \}$$

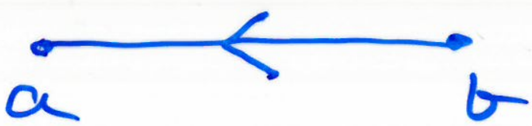
and we picture this as

The arrow is an orientation that specifies the direction of travel from a to b.

For $a < b \in \mathbb{R}$ we write

$$[b, a] = \{ x \in \mathbb{R} : a \leq x \leq b \}$$

and picture this as



We say that $[a, b]$ and $[b, a]$ are oriented intervals.

Example $S = [2,1] \cup [3,4] \cup [6,5]$

The boundary of the oriented interval $S = [a, b]$ is the set

$$\partial S = \{a, b\}$$

consisting of two points, the initial point a and final point b.

Example $S = [2,1] \cup [3,4] \cup [6,5]$

$$\partial S = \{2, 1, 3, 4, 6, 5\}$$

Terminology we'll say that $S = [a,b]$ is 1-dimensional and that ∂S is 0-dimensional

Definition Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = \underbrace{F(b)}_{\text{final point}} - \underbrace{F(a)}_{\text{initial point}}$$

Example Integrate the 0-form $\omega = 3x^2+4$ on the boundary of the interval $S = [2,1]$.

Sol^n

$$\int_{\partial S} \omega = \int_{[2,1]} 3x^2+4$$

$$= \underbrace{3(1^2)+4}_{F(b)} - \underbrace{3(2^2)+4}_{F(a)}$$

$$= 7 - 16 = -9.$$

<u>Defn</u> Given a 0-form $\omega = F(x)$ on $S = [a_1, b_1] \cup [a_2, b_2]$ we define

$$\int_{\partial S} \omega = \int_{\partial[a_1, b_1]} \omega + \int_{\partial[a_2 b_2]} \omega$$

provided

$$[a_1, b_1] \cap [a_2, b_2] = \emptyset.$$

Example Integrate the 0-form $\omega = 3x^2+4$ on ∂S where $S = [2,1] \cup [3,4]$.

Sol'n

$$\int_{\partial S} \omega = \int_{[2,1]} \omega + \int_{[3,4]} \omega$$

$$= -9 + 3(4^2)+4 - (3(3^2)+4)$$

$$= 12$$

WARNING: A 0-form can only be integrated on a 0-dimensional oriented region.