Oriented planar triangles

Three points in a plane determine a triangle.

An orientation of a triangle is specified by a curved arrow.

Corresponding to one of two possible directions of rotation.

The positive side of the oriented triangle is the one...
from which the arrow denotes anti-clockwise rotation.

An orientation is just an ordering of the vertices. The ordering PRQ denotes the second triangle above.
Constant 2-forms

Let $S$ denote an oriented triangle in $\mathbb{R}^3$.

Let $S_2$ denote the image of the triangle $S$ in the $xy$-plane under the projection

$$\rho_{xz}: \mathbb{R}^3 \to \mathbb{R}^2, \ (x,y,z) \mapsto (x,y)$$
for any constant $A \in \mathbb{R}$

let

$$\int_{S} A \, dx \, dy$$

denote

$$\pm A \times \text{(area of } S_2\text{)}$$

with sign $+1$ if

and with sign $-1$ if
Similarly define for $S, C \in R$

\[ \int_S c \, d^2\lambda \, dx \quad \text{and} \quad \int_S B \, dy \, dz \]

**Defn**

\[ \int_S \left( A \, dx \, dy \, dz + B \, dy \, dz \, dx + C \, dz \, dx \, dy \right) \]
Example Evaluate

\[ I = \int_S \mathbf{F} \cdot \hat{n} \, dA \]

over the oriented triangle \( S \) with vertices \((0,0,0), (1,2,3)\) and \((1,4,0)\) in that order.

**Solution**

\[ I = \int_S \mathbf{F} \cdot \hat{n} \, dA + \int_S 3 \, dV \]

area of \( S_2 \) \( = 1 \)
\[ \text{Area of } \triangle_{AB} = \frac{1}{2} \cdot 3 = \frac{3}{2} \]

So

\[ I = \int dx \wedge dy + 3 \int dz \wedge dx \]

\[ = \frac{1}{11} + (-1)3 \cdot \left( \frac{3}{2} \right) \]

\[ = 1 - \frac{9}{2} \]

\[ = -\frac{7}{2} \]