Partial derivatives of composite functions

\[ u = F(x_1, x_2, \ldots, x_n) \]

where

\[ x_1 = g_1(r_1, r_2, \ldots, r_p) \]
\[ x_2 = g_2(r_1, r_2, \ldots, r_p) \]
\[ \vdots \]
\[ x_n = g_n(r_1, r_2, \ldots, r_p) \]

then

\[ \frac{du}{dr_i} = \frac{du}{dx_1} \frac{dx_1}{dr_i} + \frac{du}{dx_2} \frac{dx_2}{dr_i} + \ldots + \frac{du}{dx_n} \frac{dx_n}{dr_i} \]

Proof is fairly straightforward.
Example \ Let

\[ u = x^2 e^{yx} \]
\[ x = t \cos(t) \]
\[ y = t \sin(t) \]

\text{Find}

\[ \frac{du}{dt} \text{ at } t = \frac{\pi}{2} \]

\text{Solution:}

\[ \frac{du}{dt} = \frac{du}{dx} \frac{dx}{dt} + \frac{du}{dy} \frac{dy}{dt} \]

\[ = (x^2 ye^{yx} + 2xe^{yx}) (\cos(t) - t \sin(t)) \]
\[ + x^3 e^{yx} (\sin(t) + t \cos(t)) \]

\text{Evaluate at } t = \frac{\pi}{2}\]

\text{get } \frac{du}{dt} = 0,
Proof of fundamental theorem of calculus \((n=2)\)

We want to prove

\[
\int_{s_1}^{s_2} dw = \int_{s} \frac{dw}{ds} \, ds
\]

Proof

Suppose \(w = F(x, y)\)

Suppose \(x = g(t), y = h(t)\) is a parametrization of \(s\) as \(t\) varies from \(t = t_0\) to \(t = t_1\).
\[ \int_S d\nu = \int_S \frac{\partial F}{\partial x} (g(x), h(x)) \, dx + \frac{\partial F}{\partial y} (g(x), h(x)) \, dy \]

\[ = \int_{t_0}^{t_1} \left( \frac{\partial F}{\partial x} (g(t), h(t)) \frac{g'(t)}{P} + \frac{\partial F}{\partial y} (g(t), h(t)) \frac{h'(t)}{P} \right) \, dt \]

\[ = \int_{t_0}^{t_1} \left( \frac{\partial F}{\partial t} (g(t), h(t)) \right) \, dt \]

\[ = \int_{t_0}^{t_1} \left( \frac{\partial F}{\partial t} (g(t), h(t)) \right) \, dt \]

\[ = R(g(t_0), h(t_0)) - R(g(t_1), h(t_1)) \]

\[ = R(Q_2) - R(Q_1) \]

QED
Summary of 1-forms and an introduction to 2-forms

- A 1-form is an expression such as
  \[ \omega = A \, dx + B \, dy \]
  that can be integrated over oriented curves.

- A 2-form is an expression such as
  \[ \omega = A \, dx \wedge dy + B \, dy \wedge dz + C \, dz \wedge dx \]
  that can be "integrated" over "2-dimensional oriented regions."
• Integrals of 1-forms are just limits of sums of integrals of constant 1-forms over oriented straight-line segments.

• Integrals of 2-forms are just limits of sums of integrals of "constant 2-forms" over "oriented planar triangles."