MA 2286 Differential Forms

(= Calculus)

Aim: Explain and appy the generalized Stokes formula

$$\int_{\partial S} \omega = \int_S \partial \omega$$

where

- ω is a <u>differential</u> <u>p-form</u> in n variables
- S is a <u>nice</u> region in $\mathbb{R}^n$.
- ∂S is the <u>boundary</u> of the region
- $\int$ is an <u>integral</u>.

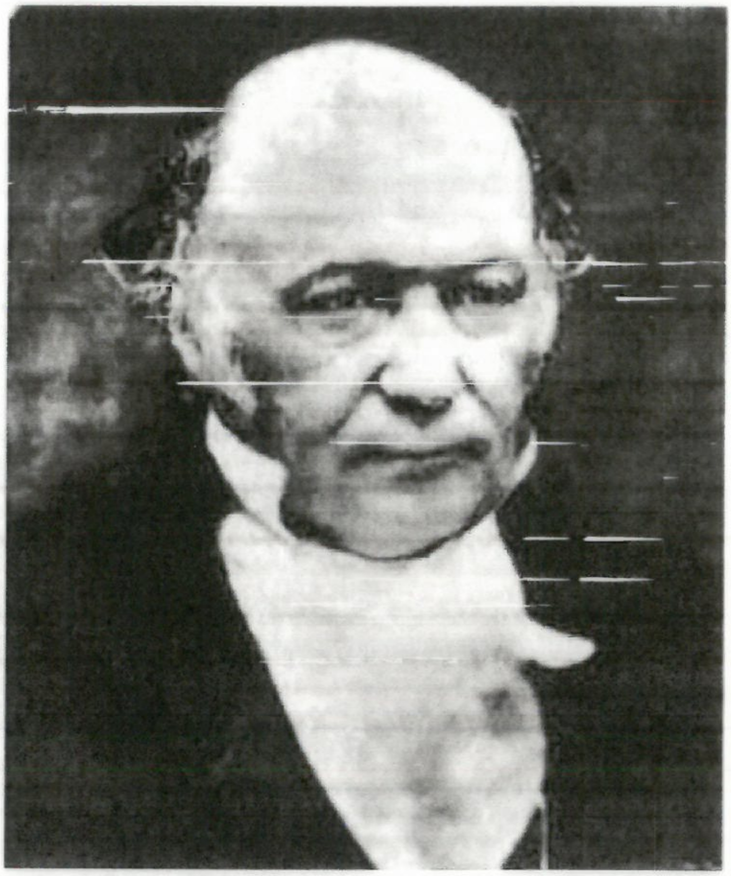
underlined words will be explained in the module.

- Born in Sligo
- Lucasian professor at Cambridge (along with Stephen Hawking, Isaac Newton, Joseph Larmor, ...)

Module covers same topics as in past years, but with a slightly different approach:

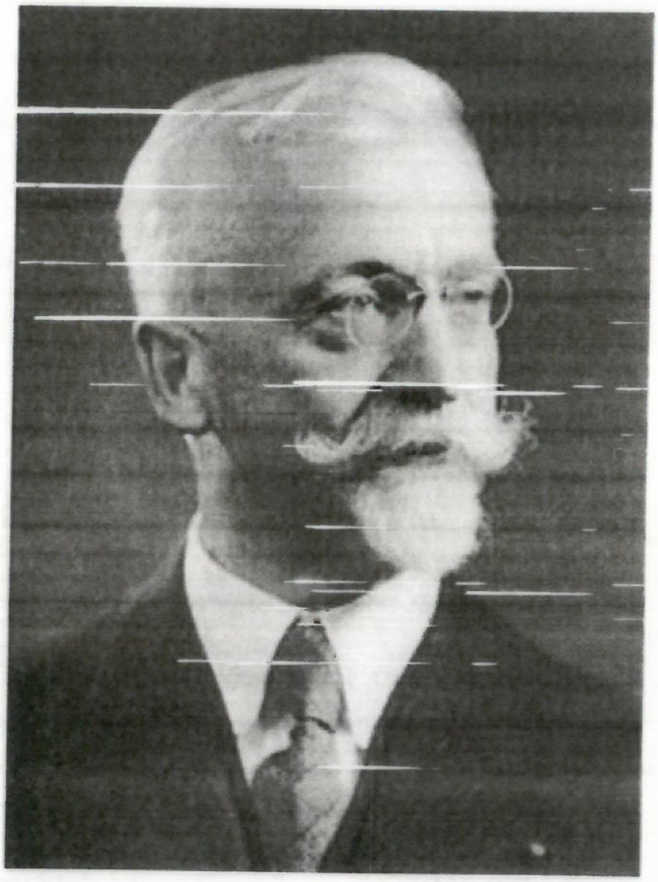
- Earlier years treated only for $p=1,2$ and only $n=2,3$. We'll treat $p \geq 0$ and $n \geq 1$.

- Previous years used vector notation of Hamilton.



Sir William Rowan Hamilton
Irish Mathematician and professor
at TCD.

We'll use Cartan's notation of differential forms in place of Hamilton's vector language.



Elie Cartan

French Mathematician.

Reason's for the "new" approach:

- Minimizes the overlap with MP231.
- Provides a simpler and unified presentation of:

- Fundamental Theorem of Calculus ($p=0$, $n=1$).
- Green's Theorem in the plane ($p=1$, $n=2$)
- Stoke's Theorem ($p=1$, $n=3$)
- Divergence Theorem (or Gauss' Theorem) ($p=2$, $n=3$).

- Differential p-forms in n variables are basic tools in modern geometry.

- "Big data" requires us to work in $\mathbb{R}^n$, n large.

e.g. A 2-dimensional colour digital image can be thought of as a function

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3, \quad \underset{\substack{\uparrow \\ \text{position} \\ \text{of pixel}}}{(x, y)} \longmapsto \underset{\substack{\uparrow \\ \text{colour} \\ \text{of pixel.}}}{(r, g, b)}$$

An MRI scan is grey scale:

$$f : \mathbb{R}^3 \longrightarrow \mathbb{R}$$

Text: Advanced Calculus
by M. Spiegel
(Schaum's Series)

Background reading:
"Advanced Calculus: a differential forms approach"
by Harold M. Edwards

Also: Spivak's book on manifolds

Continuous Assessment (30%)
Three in-class tests (each 10%)
The tests will be taken more-or-less verbatim from the homework sheet.

Final exam 70%.

Lecture notes & problems available on Blackboard.