

Example Consider

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Show that $f_x(0,0)$ exists,
 $f_y(0,0)$ exists but that $f(x,y)$
is not continuous at $(0,0)$.

Soln

$$\lim_{h \rightarrow 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$$\lim_{h \rightarrow 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0.$$

Hence $f_x(0,0) = 0$, $f_y(0,0) = 0$ exist.

Last lecture we saw

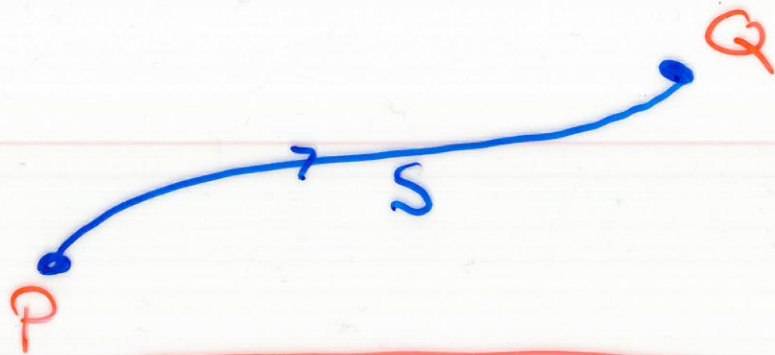
$$\lim_{\substack{x \rightarrow 0 \\ y = mx \rightarrow 0}} f(x,y) = \frac{m}{1+m^2}.$$

Since this limit depends on m , it doesn't exist. Hence $f(x,y)$ is not continuous at $(0,0)$.

Fundamental Theorem of Calculus again!

Let ω be a 0-form on n -dimensional space.

Let S be a curve in \mathbb{R}^n from P to Q .



Theorem

$$\int_S d\omega = \int_{\partial S} \omega$$

Example Evaluate

$$I = \int_S (y^3 + 2x) dx + 3xy^2 dy$$

where S is the straight line from $P = (0,0)$ to $Q = (1,2)$.

Soln (using above theorem)

Consider

$$w = xy^3 + x^2$$

Then

$$dw = (y^3 + 2x) dx + (3xy^2) dy$$

So

$$I = \int_S w = w|_Q - w|_P$$

$$= 9 - 0 = 9$$

9

Solⁿ (Using the definition of an integral)

The points

$$(x = t, y = 2t)$$

trace out the line segment

S as t goes from $t=0$, to $t=1$.

$$x = t$$

$$y = 2t$$

$$dx = dt$$

$$dy = 2 dt$$

$$I = \int_{t=0}^1 ((2t)^3 + 2t) + 3t(2t) \cdot 2 \quad dt$$

$$= \int_0^1 32t^3 + 2t \quad dt$$

$$= \frac{32t^4}{4} + t^2 \Big|_0^1 = 9.$$

Problem Evaluate

$$I = \int_S (6xy^2 - y^3) dx + (6x^2y - 3xy^2) dy$$

where S is some curve

from $P = (1, 2)$ to $Q = (3, 4)$.

Solⁿ Try to find $w = F(x, y)$

such that

$$dw = F_x dx + F_y dy$$

$$= (6xy^2 - y^3) dx + (6x^2y - 3xy^2) dy.$$

well

$$F(x, y) = 3x^2y^2 - xy^3 + g(y)$$

$$F(x, y) = 3x^2y^2 - xy^3 + h(x)$$

We conclude that $g(y)$ and

$h(x)$ are equal to some constant C .

SG

$$I = \int_S d(3x^2y^2 - xy^3 + c)$$

$$\stackrel{=}{\text{PTC}} \int_{\partial S} 3x^2y^2 - xy^3 + c$$

$$= 3x^2y^2 - xy^3 + C \Big|_{(1,2)}^{(3,4)}$$

A hand-drawn diagram of a cell. It features a large, irregular outer boundary representing the cell membrane. Inside this boundary is a smaller, roughly circular structure representing the nucleus. Within the nucleus is a small, dense, dark-shaded circle representing the nucleolus. The entire drawing is done in blue ink on a white background.

$$= 236$$
