0-forms on n-dimensional space

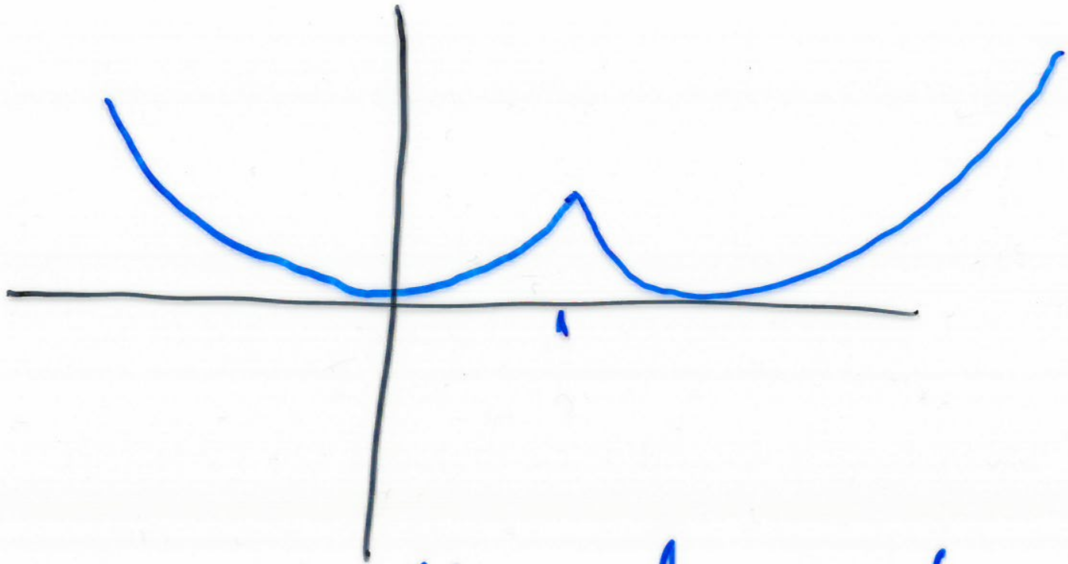
A differential 0-form on 2-dimensional space is a real valued function

$$\omega = f(x, y)$$

which is "differentiable". To explain this term, recall:

Informally A function $f(x)$ is differentiable at a point x if the curve $y = f(x)$ has a well-defined tangent line at x.

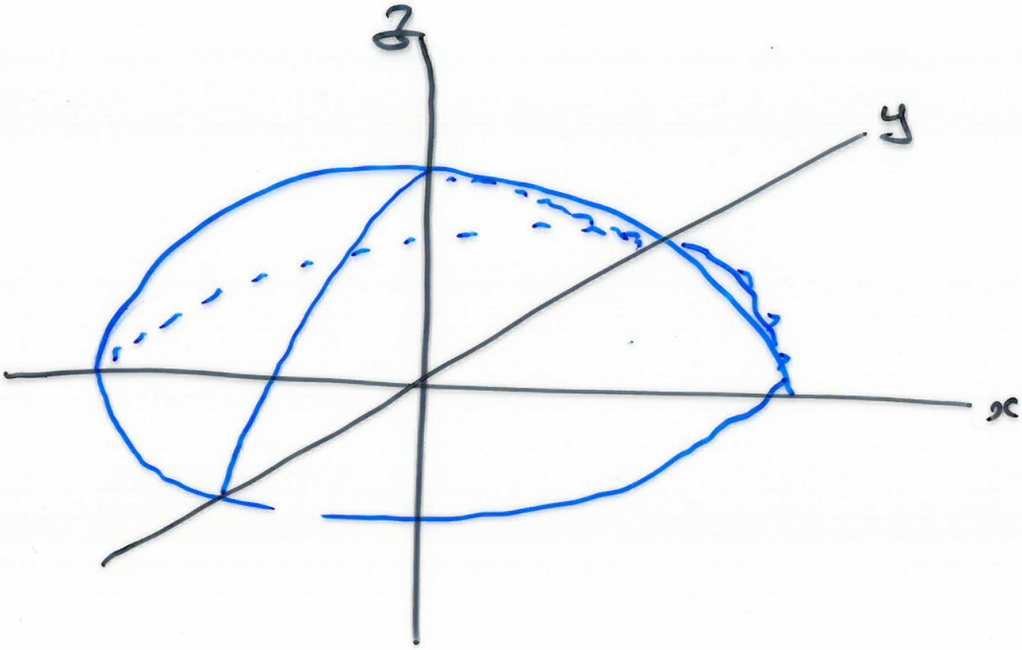
Example
$$y = \begin{cases} x^2 & x \leq 1 \\ (x-2)^2 & x > 1 \end{cases}$$



Not differentiable at $x=1$.

Informally A function $f(x,y)$ is ~~x~~ differentiable at a point (x,y) if the surface $z=f(x,y)$ has a well-defined tangent plane at (x,y).

Example $z=\sqrt{1-x^2-y^2}$ is defined for all $x^2+y^2\le 1$, and is the surface



For any (x,y) in

$$S = \{(x,y) : x^2+y^2 \leq 1\}$$

the surface has a tangent plane. So

$$\omega = \sqrt{1-x^2-y^2}$$

is a differential 0-form on S.

We'll skip the formal definition of differentiability.

Differential 1-forms on n-dimensional space

A differential 1-form on a 2-dimensional space is a function

$$\omega = A(x,y)\, h_1 + B(x,y)\, h_2$$

that inputs a vector (x,y) and a vector (h_1, h_2) and returns a real number. Here $A(x,y)$ and $B(x,y)$ must be differentiable functions.

<u>Example</u> Evaluate the 1-form

$$\omega = (x^2+y^2)\, h_1 + 2xy\, h_2$$

at $(x,y) = (2,4)$ and $(h_1, h_2) = (\frac{1}{4}, \frac{1}{4})$.

Sol^n

$$(4+16)\,0.25 + 2.2.4.(0.25)$$

$$= 5+4 = 9.$$

Notation We usually denote

$$\omega = A(x,y)\,h_1 + B(x,y)\,h_2$$

by

$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

Example Evaluate the 1-form

$$\omega = (x^2+y^2)\,dx + 2xy\,dy$$

at $(x,y)=(2,4)$, $(h_1,h_2)=(\frac{1}{4},\frac{1}{4})$

Sol^n

9

<u>Example</u> A particle is moved in a constant force field. It takes 3 units of work to move the particle from point (x, y) to point $(x+1, y)$. It takes 4 units of work to move the particle from (x, y) to the point $(x, y+1)$. We say that the work is represented by the 1-form

$$\omega = 3\,dx + 4\,dy$$

<u>Example</u> Consider a particle in a constant force field, with work given by the 1-form

$$\omega = 2dx + 3dy + 5dz$$

Calculate the work done by moving the particle along the straight-line segment from point $P = (-1, 3, -5)$ to point $Q = (3, -1, -7)$.

<u>Sol^{n}</u>

$Q - P = (3,-1,-7) - (-1,3,-5)$

$= (4,-4,-2)$

work $= 2(4) + 3(-4) + 5(-2) = -14$ units.

or

work $(2,3,5) \cdot (4,-4,-2) = -14$.