Differential 0-forms in 1 variable

A differentiable 0-form in 1-variable is just a differentiable, real-valued function

$$\omega = f(x)$$

Example

$$\omega = 3x + 4$$
$$\omega = 3x^2 + 4$$
$$\omega = \sin(x)$$

are all differentiable 0-forms.

Usually, a diff. 0-form is given in the context of some closed interval

$$S = [a, b] \subseteq \mathbb{R},$$

or union of closed intervals

$$S = [a_1, b_1] \cup [a_2, b_2] \cup \cdots \cup [a_k, b_k].$$

We only require ω to be differentiable on the points in S.

Example

$$\omega = |x|$$

is a differentiable 0-form on $S = [1, 100]$.

Clearly ω is not a differentiable 0-form on the set

$$S = [-100, 100]$$

Terminology We'll say 0-form to mean differentiable 0-form.

For $a < b \in \mathbb{R}$ we write

$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and we picture this as

The arrow is an orientation

which specifies that the direction of travel is from a to b.

For $a < b \in \mathbb{R}$ we write

$$[b, a] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and picture this as

In this case, the direction of travel starts at b and ends at a.

We say that $[a, b]$ and $[b, a]$ are oriented intervals.

Example $S = [2,1] \cup [3,4] \cup [6,5]$

The **boundary** of the interval $S = [a, b]$ is the set

$$\partial S = \{a, b\}$$

consisting of two points, the initial point a and the final point b.

Example $S = [2,1] \cup [3,4] \cup [6,5]$

$$\partial S = \{1, 6, 2, 3, 5, 4\}$$

Terminology: we'll say that $S = [a, b]$ is 1-dimensional, and that ∂S is 0-dimensional

Definition Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = \underbrace{F(b)}_{\text{final point}} - \underbrace{F(a)}_{\text{initial point}}$$

Example Integrate the 0-form $\omega = 3x^2+4$ on the boundary ∂S of the set $S = [2,1]$.

Sol^n

$$\int_{\partial S} \omega = \int_{\{2,1\}} 3x^2+4$$

$$= \underbrace{3(1^2)+4}_{F(b)} - \underbrace{3(2^2)+4}_{F(a)}$$

$$= 7 - 16 = -9$$

Def^n Given a 0-form $\omega = F(x)$ on $S = [a_1,b_1] \cup [a_2,b_2]$ we define

$$\int_{\partial S} \omega = \int_{\partial [a_1,b_1]} \omega + \int_{\partial [a_2,b_2]} \omega$$

provided that

$$[a_1, b_1] \cap [a_2, b_2] = \emptyset.$$

<u>Example</u> integrate the 0-form $\omega = 3x^2+4$ on ∂S where $S = [2,1] \cup [3,4]$.

<u>Sol'n</u>

$$\int_{\partial S} \omega = \int_{[2,1]} \omega + \int_{[3,4]} \omega$$

$$= -9 + 3(4^2)+4 - (3(3^2)+4)$$

$$= 12$$

WARNING: A 0-form can only be integrated over a 0-dimensional oriented region.