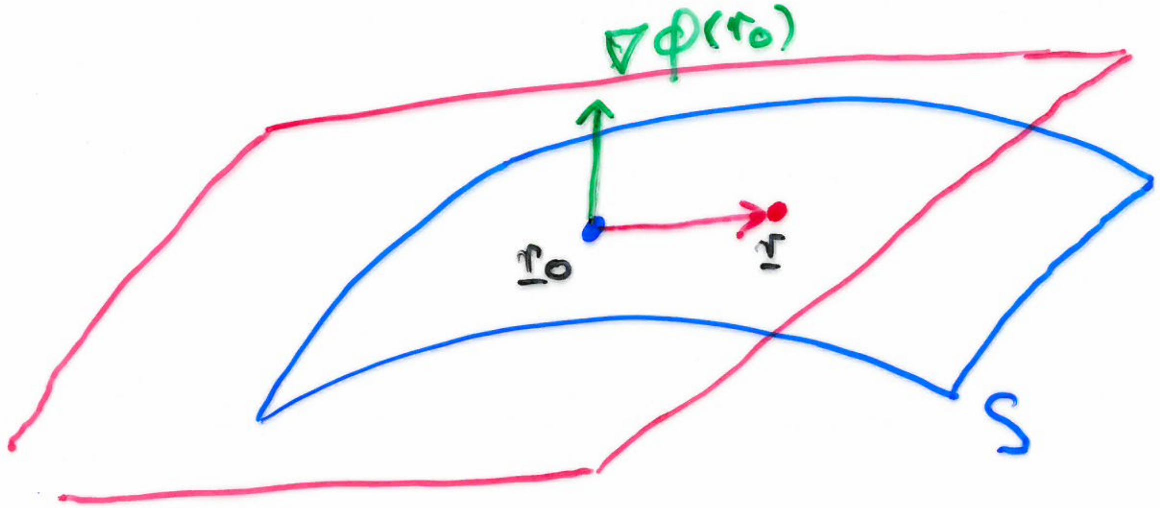

The tangent plane to S at some point

$$\underline{r}_0 = (x_0, y_0, z_0),$$

where S is described by

$$\phi(x, y, z) = k,$$

consists of all points $\underline{r} = (x, y, z)$ such that

$$\boxed{\nabla\phi(\underline{r}_0) \cdot (\underline{r} - \underline{r}_0) = 0}$$

<u>Example</u> Find a unit normal vector to the surface S:

$$2x^2 + 4yz - 5z^2 = -10$$

at the point $\underline{r}_0 = (3, -1, 2)$. Then find the equation of the tangent plane to S at $(3, -1, 2)$.

<u>Soln</u> Note

$$2(3^2) + 4(-1)(2) - 5(2^2) = -10$$

Let

$$\phi(x, y, z) = 2x^2 + 4yz - 5z^2$$

$$\text{grad}\,\phi = \nabla\phi = \frac{\partial\phi}{\partial x}\underline{i} + \frac{\partial\phi}{\partial y}\underline{j} + \frac{\partial\phi}{\partial z}\underline{k}$$

$$= 4x\,\underline{i} + 4z\,\underline{j} + (4y - 10z)\,\underline{k}$$

$$\nabla\phi(\underline{r}_0) = 12\,\underline{i} + 8\,\underline{j} - 24\,\underline{k}$$

$$= (12, 8, -24)$$

So $(12, 8, -24)$ is normal (i.e. at right angles) to S at the point $\underline{r}_0 = (3, -1, 2)$.

A unit normal vector must have Length 1.

$$\|(12, 8, -24)\| = \sqrt{12^2 + 8^2 + (-24)^2} = 28$$

So a unit normal vector is

$$\underline{n} = \frac{1}{28}(12, 8, -24) = \frac{1}{14}(6, 4, -12)$$

$$\underline{n} = \frac{1}{7}(3, 2, -6)$$

The equation of the tangent plane is

$$(3, 2, -6) \cdot ((x, y, z) - (3, -1, 2)) = 0$$

$$(3, 2, -6) \cdot (x-3, y+1, z-2) = 0$$

$$3x - 9 + 2y + 2 - 6z + 12 = 0$$

$$\boxed{3x + 2y - 6z = -5}$$

Required equation of the tangent plane.

Curl

Given a vector field

$$F = F_1 \underline{i} + F_2 \underline{j} + F_3 \underline{k}$$

on $\mathbb{R}^3$ (where

$F_1 = F_1(x, y, z)$ real valued function

$F_2 = F_2(x, y, z)$ "

$F_3 = F_3(x, y, z)$ ")

we define the associated work 1-form

$$\omega = F_1 dx + F_2 dy + F_3 dz .$$

Suppose the exterior derivative of ω is

$$d\omega = A\, dx \wedge dy + B\, dy \wedge dz + C\, dz \wedge dx .$$

Then we define

$$\mathrm{Curl}(F) = B\underline{\underline{i}} + C\underline{\underline{j}} + A\underline{\underline{k}}.$$

<u>Example</u> $F = xz\,\underline{\underline{i}} + y^2\,\underline{\underline{j}} + 2x^2y\,\underline{\underline{k}}$

Find $\mathrm{Curl}(F)$.

$$\omega = xz\,dx + y^2\,dy + 2x^2y\,dz$$

$$d\omega = (z\,dx + x\,dz)\wedge dx$$
$$+ (2y\,dy)\wedge dy$$
$$+ (4xy\,dx + 2x^2dy)\wedge dz$$

$$= \overbrace{\cancel{z\,dx\wedge dx}}^{=0} + x\,dz\wedge dx$$
$$+ \overbrace{\cancel{2y\,dy\wedge dy}}^{=0}$$
$$+ 4xy\,dx\wedge dz + 2x^2dy\wedge dz$$

$$= x\,dz\wedge dx - 4xy\,dz\wedge dx + 2x^2dy\wedge dz$$

$$= (x - 4xy)\,dz\wedge dx + 2x^2\,dy\wedge dz$$

So

$$\mathrm{Curl}(F) = (x - 4xy)\,\underline{\underline{j}} + 2x^2\,\underline{\underline{i}}$$

or

$$\mathrm{Curl}(F) = (2x^2,\ x - 4xy,\ 0).$$