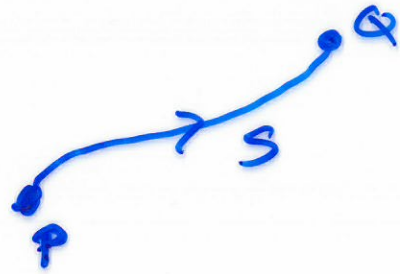
To prove the FTC

$$\int_S d\omega = \int_{\partial S} \omega$$



we need a bit about:

Partial Derivatives of Composite Functions

If

$$u = F(x_1, x_2, \dots, x_n)$$

where

$$x_1 = g_1(r_1, r_2, \dots, r_p)$$

$$x_2 = g_2(r_1, r_2, \dots, r_p)$$

$$\vdots$$

$$=$$

$$x_n = g_n(r_1, r_2, \dots, r_p)$$

then

$$\frac{\partial u}{\partial r_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial r_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial r_i} + \frac{\partial u}{\partial x_3}\frac{\partial x_3}{\partial r_i} + \dots$$

$$\dots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial r_i} \; .$$

Example Let

$$u = x^2 e^{yx}$$

$$x = t\cos(t)$$

$$y = t\sin(t)$$

Find $\frac{\partial u}{\partial t}$ at $t = \frac{\pi}{2}$

Sol^n

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

$$= \left(yx^2 e^{yx} + 2x e^{yx}\right)\left(-t\sin t + \cos t\right)$$

$$+ \left(x^3 e^{yx}\right)\left(t\cos t + \sin t\right)$$

Evaluate at $t = \frac{\pi}{2}$ to get

$$-\frac{\pi^3}{8}.$$

Proof of Fundamental Theorem of Calculus $(n=2)$

Suppose $\omega = F(x,y)$

suppose $x = g(t)$, $y = h(t)$ is a parametrization of S as t goes from t_0 to t_1

$$\int_S d\omega = \int_S F_x \, dx + F_y \, dy$$

$$= \int_{t_0}^{t_1} F_x(g(t), h(t)) \frac{\partial g}{\partial t} dt + F_y(g(t), h(t)) \frac{\partial h}{\partial t} dt$$

$$= \int_{t_0}^{t_1} \frac{\partial F}{\partial t} dt = F(g(t_1), h(t_1)) - F(g(t_0), h(t_0))$$

$$= F(Q) - F(P)$$

$$= \int_{\partial S} \omega \, .$$

QED

Summary of 1-forms and an introduction to 2-forms.

- A 1-form is an expression such as

$$\omega = A\,dx + B\,dy$$

that can be integrated over oriented curves.

- A 2-form is an expression such as

$$\omega = A\,dx \wedge dy + B\,dy \wedge dz + C\,dz \wedge dx$$

that can be "integrated" over "2-dimensional oriented regions".

- integrals of 1-forms are just limits of sums of integrals of constant 1-forms over

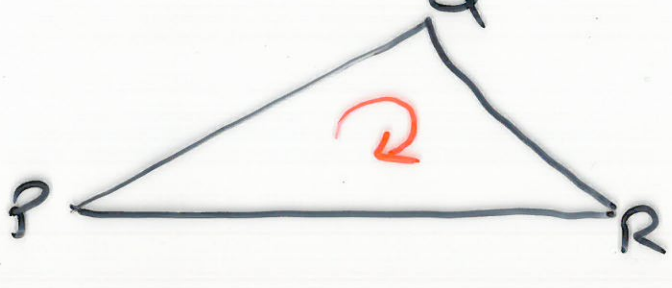
oriented straight-line segments.

- integrals of 2-forms are just limits of sums of integrals of constant 2-forms over oriented planar triangles.

Oriented planar triangles

Three points in a plane determine a triangle

An orientation of a triangle is specified by a curved arrow



corresponding to one of two direction of rotation.

The positive side of the oriented triangle is the one from which the arrow denotes anti-clockwise rotation.

An orientation is just an ordering of the vertices. The ordering PRQ denotes the second triangle above.