MA286 Analysis (= calculus)

Aim: Explain and apply the generalised Stokes formula

$$\int_{\partial S} \omega = \int_S d\omega$$

where:

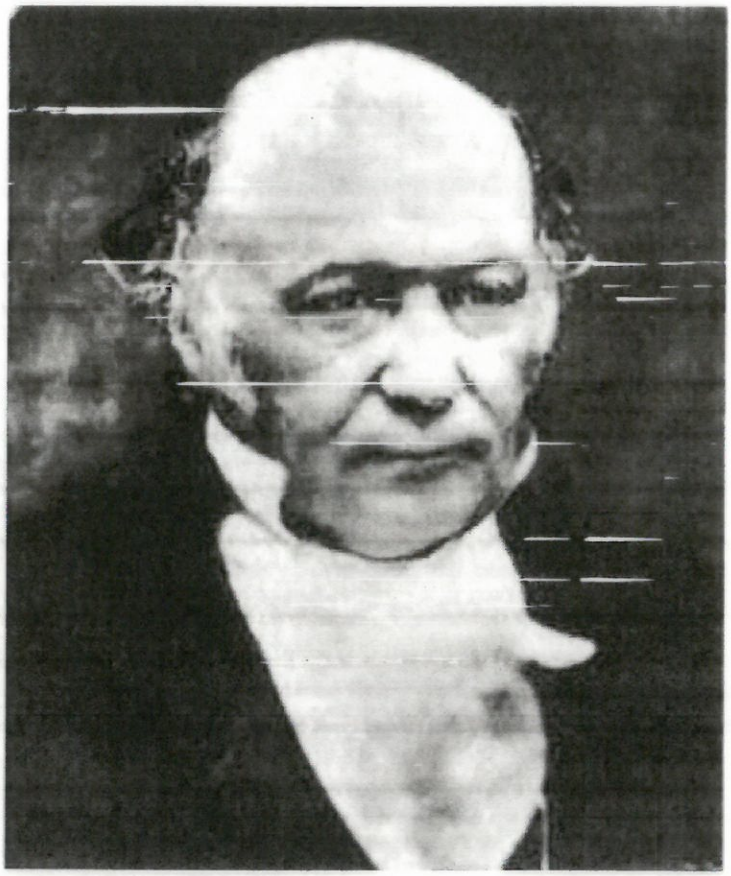
- ω is a differential p-form in n variables
- S is a nice region in $\mathbb{R}^n$.
- ∂S is the boundary of the region.
- $\int$ is an integral

Underlined words will be explained in course.

- Born in Sligo
- Lucasian professor at Cambridge (along with Stephen Hawking, Isaac Newton, Joseph Larmor, ...)

Course covers same topic as in earlier years but with a different approach:

- Earlier years treated only $p = 1, 2$ and only $n = 2, 3$. We'll treat $p \geq 0$, and $n \geq 1$.
- Previous years used vector notation of Hamilton.



Sir William Rowan Hamilton
Irish Mathematician and professor
at TCD.

- We'll use Cartan's notion of differential form in place of Hamilton's vector language.

Elie Cartan

French Mathematician.

Reasons for replacing Hamilton by Cartan:

- Minimizes the overlap with MP231.
- Provides a simpler and unified presentation of:
  - Fundamental Theorem of Calculus ($p=0, n=1$)
  - Green's Theorem in plane ($p=1, n=2$)
  - Stokes Theorem ($p=1, n=3$)
  - Divergence Theorem (or Gauss Theorem) ($p=2, n=3$).

- Differential p-forms in n variables are basic tools in modern geometry.

- "Big data" requires us to work in $\mathbb{R}^n$, n large.

e.g. A 2-dimensional colour digital image can be thought of as a function

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3, \quad (x,y) \longmapsto (r,g,b)$$

(x,y): position of pixel; (r,g,b): colour of pixel.

An MRI scan in grey scale:

$$f : \mathbb{R}^3 \longrightarrow \mathbb{R}, \quad (x,y,z) \longrightarrow f(x,y,z)$$

$f(x,y,z)$: grey scale.

Text: Advanced Calculus
by M. Spiegel
(Schaum's series)

Background reading:
Advanced Calculus: a
differential forms approach
by Harold M. Edwards.

Continuous Assessment (30%):
Three in-class tests (each 10%)

Final exam 70%.

Test questions will be taken
more-or-less verbatim from
the homework sheet.

Lecture notes & problems
available from Blackboard.