Given a 0-form

$$\omega = F(x)$$

we define its exterior derivative (or derivative) to be the 1-form

$$d\omega = F'(x)\,dx$$

Recall from 1st year that substitutions are useful for calculating integrals.

Example Find a differential 0-form ω whose derivative $d\omega$ is

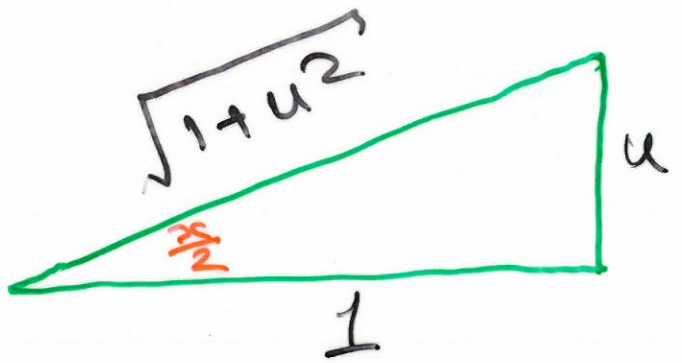
$$d\omega = \frac{1}{5+3\cos(x)}\,dx$$

Sol^n using the language of 1^{st} year, we want to find

$$w = \int \frac{1}{5+3\cos(x)}\,dx$$ (indefinite integral)

Sol^n

Let $u = \tan\frac{x}{2}$



$\sin\frac{x}{2} = \frac{u}{\sqrt{1+u^2}}$ $\cos\frac{x}{2} = \frac{1}{\sqrt{1+u^2}}$

$$\cos(x) = \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$= \frac{1}{1+u^2} - \frac{u^2}{1+u^2} = \frac{1-u^2}{1+u^2}$$

$$du = \frac{1}{2}\sec^2\frac{x}{2}\ dx$$

$$dx = 2\cos^2\left(\frac{x}{2}\right)\,du$$

$$= \frac{2}{1+u^2}\,du$$

So

$$w = \int \frac{1}{5+3\cos(x)}\,dx$$

$$= \int \frac{1}{5+3\left(\frac{1-u^2}{1+u^2}\right)} \cdot \frac{2}{1+u^2}\,du$$

$$= \ldots$$

$$= \int \frac{du}{4+u^2}$$

from log book

$$w = \frac{1}{2}\tan^{-1}\frac{u}{2}$$

$$w = \frac{1}{2}\tan^{-1}\left\{\tan\left(\frac{x}{2}\right)\right\}$$

0-forms on n-dimensional space

A differential 0-form on 2-dimensional space is a real valued function

$$\omega = f(x, y)$$

which is "differentiable".

To explain this term informally recall from 1st year:

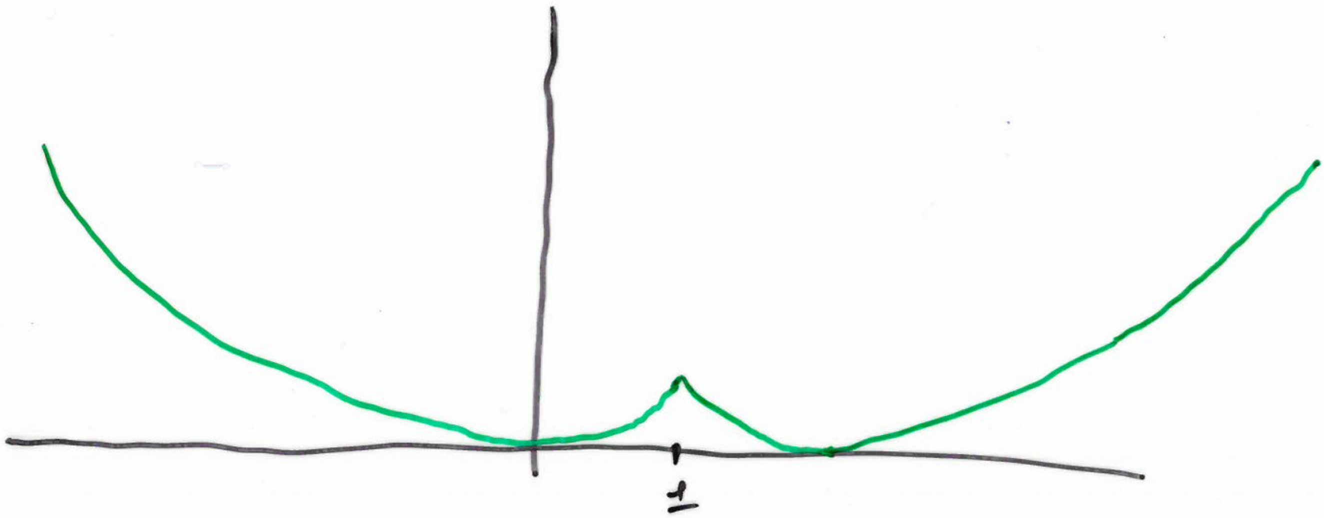
Informally A function $f(x)$ is differentiable at a point x if the curve

$$y = f(x)$$

has a well-defined tangent line at x.

<u>Example</u> Consider

$$f(x) = \begin{cases} x^2, & x \le 1 \\ (x-2)^2, & x > 1 \end{cases}$$



not differentiable at $x = 1$.

<u>Informally</u> A function

$$f(x, y)$$

is differentiable at a point (x, y) if the surface

$$z = f(x, y)$$

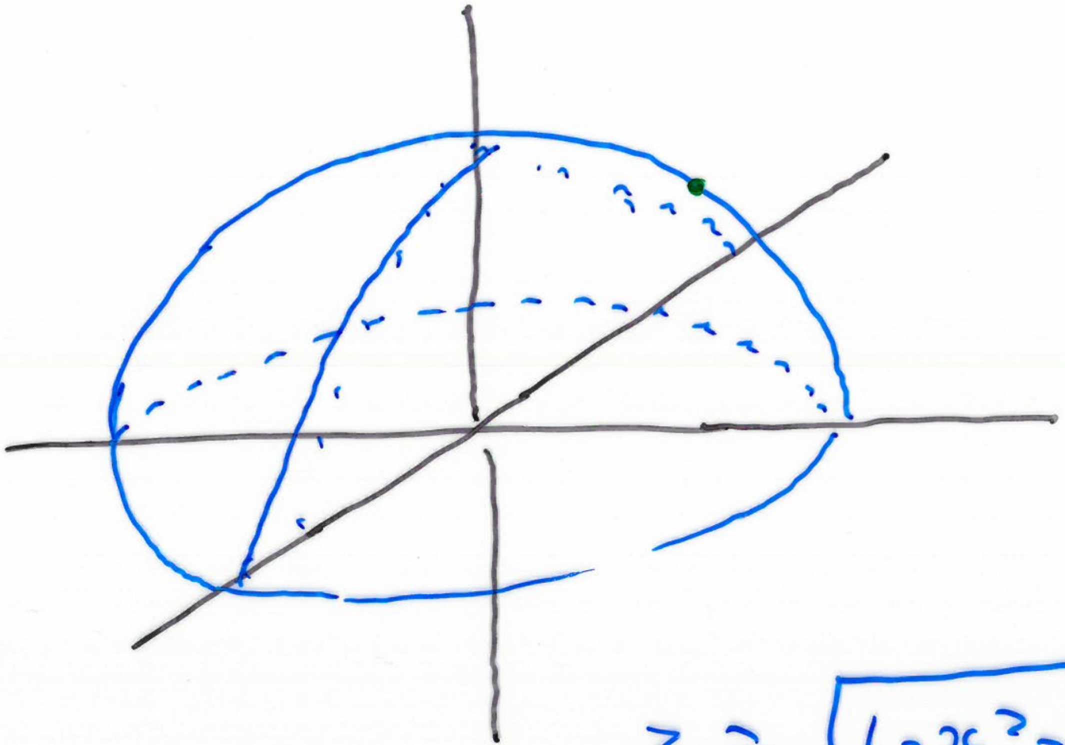
has a well-defined tangent plane at the point (x, y).

<u>Example</u> Consider

$$f(x,y) = \sqrt{1 - x^2 + y^2}$$

defined on the region

$$S = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1 \}$$



$$z = \sqrt{1 - x^2 - y^2}$$

For any $(x,y) \in S$ this surface has a well-defined tangent plane. So it is differentiable.