Differential 0-forms in 1 variable

A **differential** **0-form** in 1 variable is just a differentiable real valued function

$$\omega = f(x) .$$

Examples

$$\omega = 3x + 4$$

$$\omega = 3x^2 + 4$$

$$\omega = \sin(x)$$

usually a diff. 0-form is given in the context of some closed interval

$$S = [a, b] \subseteq \mathbb{R}$$

or union of closed intervals

$$S = [a_1, b_1] \cup [a_2, b_2] \cup \dots \cup [a_k, b_k]$$

We only require ω to be differentiable at each point in S.

<u>Example</u> $\omega = |x|$ is a differentiable 0-form on $S = [1, 100]$ say.

Clearly ω is <u>not</u> differentiable at $x=0$, but $0 \notin S$.

<u>Terminology</u> We'll write

0-form

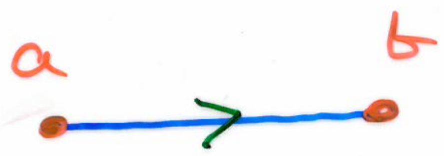
instead of

differential 0-form

For $a < b \in \mathbb{R}$ we write

$$[a,b] = \{x \in \mathbb{R} : a \le x \le b\}$$

and picture this as

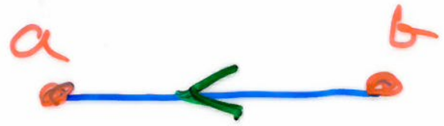


The arrow is an orientation which specifies that the direction of travel is from a to b. We say a is the initial point and b is the final point

For $a < b \in \mathbb{R}$ we write

$$[b, a] = \{x \in \mathbb{R} : a \le x \le b\}$$

and picture this as



In this case the direction of travel is deemed to start at b and end at a.

We say that $[a, b]$ and $[b, a]$ are <u>oriented intervals</u>.

<u>Example</u> $[2, 1] \cup [3, 4] \cup [6, 5]$ is a union of oriented intervals

The <u>boundary</u> of the oriented interval

$$S = [a, b]$$

is the set

$$\partial S = \{a, b\}$$

the initial point a and final point b.

<u>Terminology</u> We say that

$$S = [a, b]$$

is <u>1-dimensional</u>, and that $\partial S = \{a, b\}$ is <u>0-dimensional</u>.

Definición Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b].$$

We define the integral

$$\int_{\partial S} \omega = \underset{\substack{\uparrow \\ \text{final} \\ \text{point}}}{F(b)} - \underset{\substack{\uparrow \\ \text{initial} \\ \text{point}}}{F(a)}$$

Example Integrate the 0-form

$$\omega = 3x^2 + 4$$

on the boundary ∂S of

$$S = [2, 1].$$

Sol^n

$$\int_{\partial S} 3x^2 + 4 = \underbrace{3(1^2)+4}_{F(1)} - \underbrace{\left[3(2^2)+4\right]}_{F(2)}$$

$$= -9$$

__Defn__ Given a 0-form
$\omega = F(x)$ on

$$S = [a_1, b_1] \cup [a_2, b_2]$$

we define

$$\int_{\partial S} \omega = \int_{\partial [a_1, b_1]} \omega + \int_{\partial [a_2, b_2]} \omega$$

provided $[a_1, b_1] \cap [a_2, b_2] = \emptyset$.

__Example__ Integrate the 0-form
$\omega = 3x^2 + 4$ on ∂S
where $S = [2,1] \cup [3,4]$.

__Sol^n__

$$\int_{\partial S} \omega = \int_{\partial [2,1]} \omega + \int_{\partial [3,4]} \omega$$

$$= -9 + 3(4^2) + 4 - 3(3^2) - 4$$

$$= -9 + 3 \times 7 = 12.$$