Last time

For a function $\phi(x,y,z)$ we have

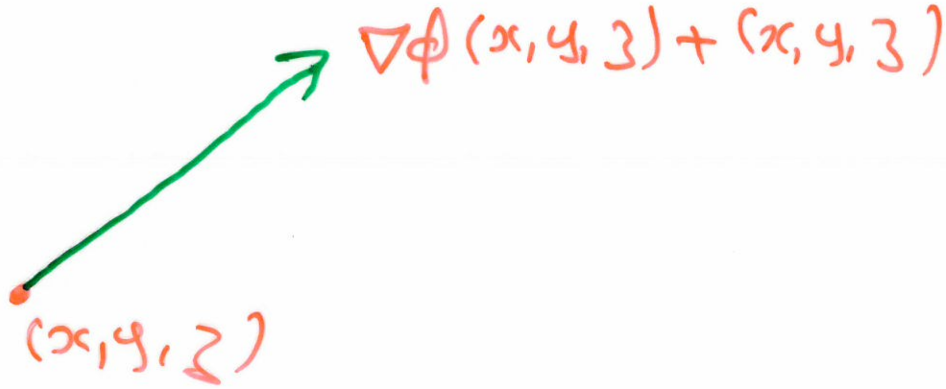
$$\text{grad}\,\phi = \nabla\phi = \frac{\partial\phi}{\partial x}\underline{\underline{i}} + \frac{\partial\phi}{\partial y}\underline{\underline{j}} + \frac{\partial\phi}{\partial z}\underline{\underline{k}}$$

Example $\phi(x,y,z) = x^2+y^2+z^2$

$$\nabla\phi = 2x\,\underline{\underline{i}} + 2y\,\underline{\underline{j}} + 2z\,\underline{\underline{k}}$$

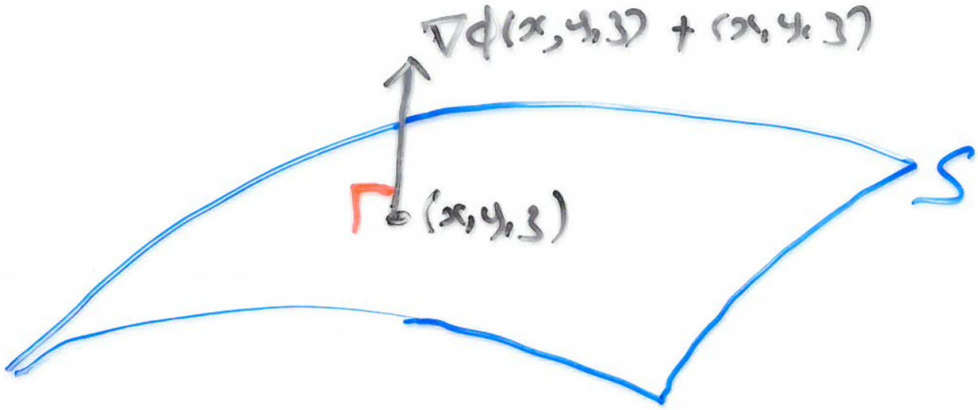
$$= \underline{\underline{(2x, 2y, 2z)}}$$

We picture $\nabla\phi$ as a "vector field" one vector at each point (x,y,z).



Last time

For a surface $\phi(x,y,z) = k$ the vector $\nabla\phi$ is always perpendicular to the surface.

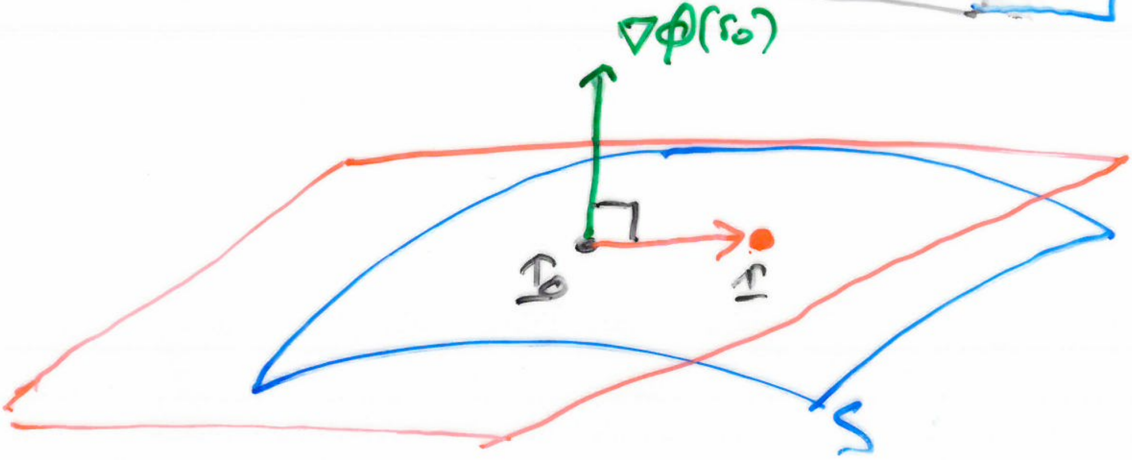


Observation The tangent plane to S at some point

$$\underline{r}_0 = (x_0, y_0, z_0)$$

consists of all points $\underline{r} = (x, y, z)$ such that

$$\boxed{\nabla\phi(\underline{r}_0) \cdot (\underline{r} - \underline{r}_0) = 0}$$



__Example__ Find a unit normal vector to the surface S:

$$2x^2 + 4yz - 5z^2 = -10$$

at the point $(3,-1,2)$.

__Sol^n__ Note

$$2.3^2 + 4(-1)(2) - 5.2^2 = -10$$

so $(3,-1,2)$ does lie on S.

Let $\phi(x,y,z) = 2x^2 + 4yz - 5z^2$

$$\nabla\phi = 4x\,\underline{\underline{i}} + 4z\,\underline{\underline{j}} + (4y - 10z)\,\underline{\underline{k}}$$

$$= (4x, 4z, 4y - 10z)$$

At $(3,-1,2)$

$$\nabla\phi = (12, 8, -24) \leftarrow$$ normal vector to S

$$\sqrt{12^2 + 8^2 + (-24)^2} = 7$$

Unit normal vector to S at $(3,-1,2)$ is: $\underline{\underline{n}} = \frac{1}{7}(12, 8, -24)$

<u>Example</u> Find the equation of the tangent plane to the surface

$$2x^2 + 4yz - 5z^2 = -10$$

at $(3,-1,2)$.

<u>Sol^n</u> The equation is

$$(12, 8, -24) \cdot \left((3,-1,2) - (x,y,z) \right) = 0$$

$$(12, 8, -24) \cdot (3-x, -1-y, 2-z) = 0$$

$$36 - 12x - 8 - 8y - 48 + 24z = 0$$

$$\boxed{-12x - 8y + 24z = 20}$$

Equation of tangent plane to S at $(3,-1,2)$.

Curl

Let $F_1(x,y,z)$, $F_2(x,y,z)$, $F_3(x,y,z)$

be three differentiable functions

of x, y, z.

Consider the vector field

$$\underline{F} = F_1 \underline{i} + F_2 \underline{j} + F_3 \underline{k}$$

which assigns to each point

$(x,y,z) \in \mathbb{R}^3$ the vector/arrow

$\underline{F}(x,y,z)$.

One defines $\mathrm{curl}(\underline{F})$ to be

the vector value function

$\mathrm{curl}(\underline{F}) =$

$$\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\underline{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\underline{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\underline{k}$$

<u>Example</u> Suppose

$$F = xy\,\underline{\underline{i}} - y^2\,\underline{\underline{j}} + 2x^2y\,\underline{\underline{k}}$$

$$\mathrm{Curl}(F) = 2x^2\,\underline{\underline{i}} - 4xy\,\underline{\underline{j}} - x\,\underline{\underline{k}}$$

we can think of the vector field

$$F = F_1\,\underline{\underline{i}} + F_2\,\underline{\underline{j}} + F_3\,\underline{\underline{k}}$$

as a 1-form

$$\omega = F_1\,dx + F_2\,dy + F_3\,dz$$

This is called the <u>work</u> <u>form</u> associated to F.

now

$d\omega = \ldots$ Next lecture!