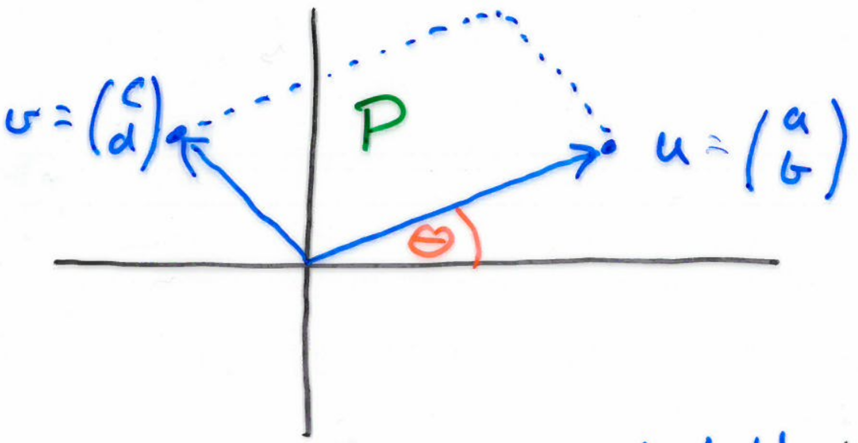
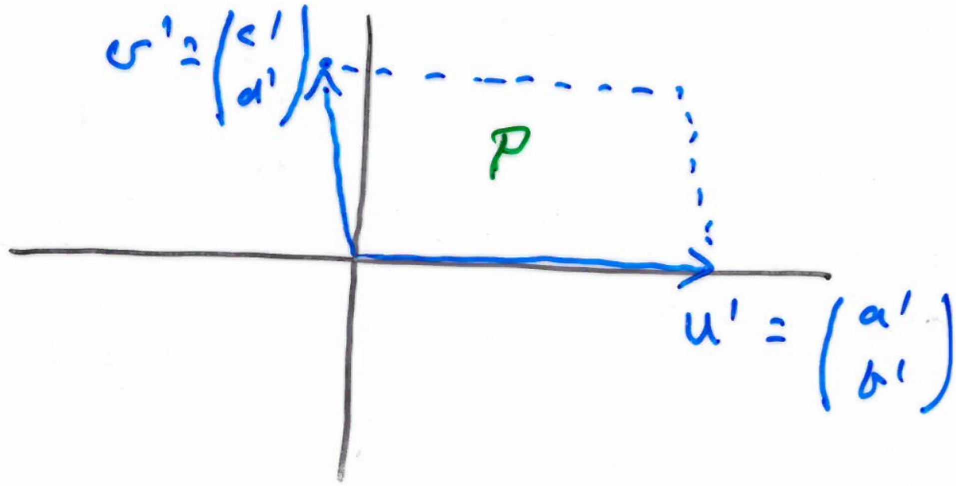
Area of a parallelogram





Prop^n Area of $P = \pm \begin{vmatrix} a & c \\ b & d \end{vmatrix} = \pm(ad - bc)$

Proof

$$u = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} u'$$

$$v = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} v'$$

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} =$$

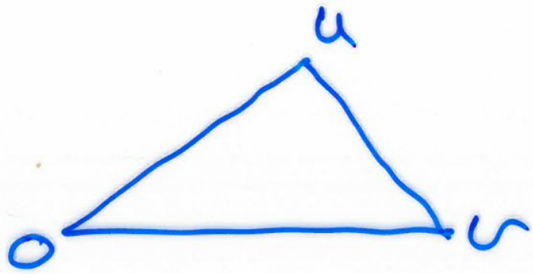
$$\det\left(\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} a' & c' \\ 0 & d' \end{pmatrix}\right) =$$

$$= \det\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \det\begin{pmatrix} a' & c' \\ 0 & d' \end{pmatrix}$$

$$= a'd'$$

$= \pm$ area of P.

<u>Corollary</u> Area of the triangle



is $\pm\frac{1}{2}\det\begin{pmatrix} | & | \\ u & v \\ | & | \end{pmatrix}$.

Example

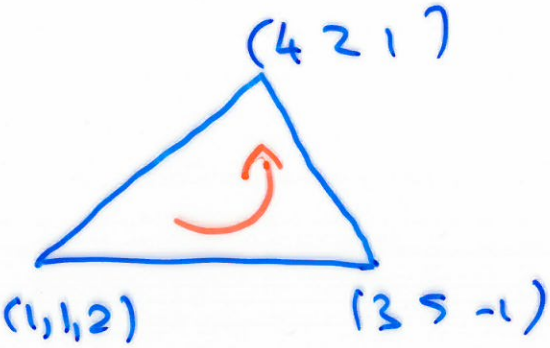
Evaluate

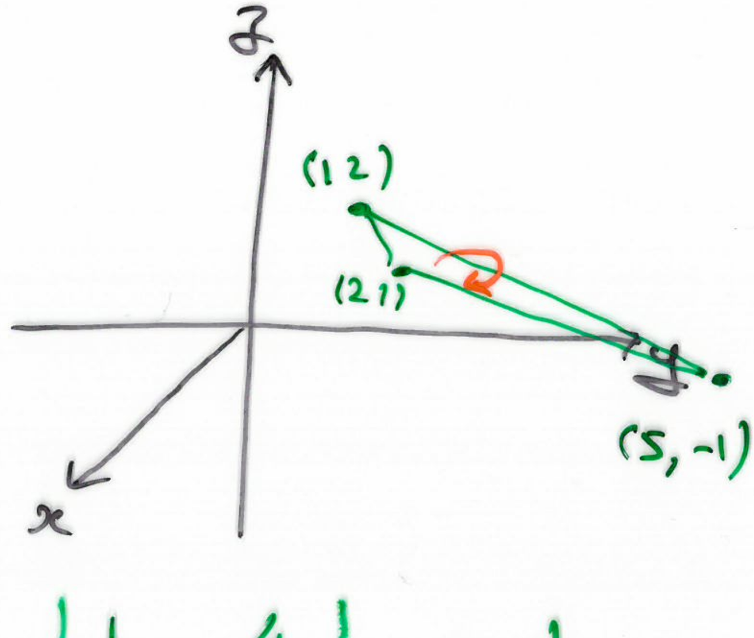
$$I = \int_S dy \wedge dz + dz \wedge dx + dx \wedge dy$$

where S is the oriented triangle with vertices $(1,1,2)$, $(3,5,-1)$, $(4,2,1)$ in that order.

Sol^n

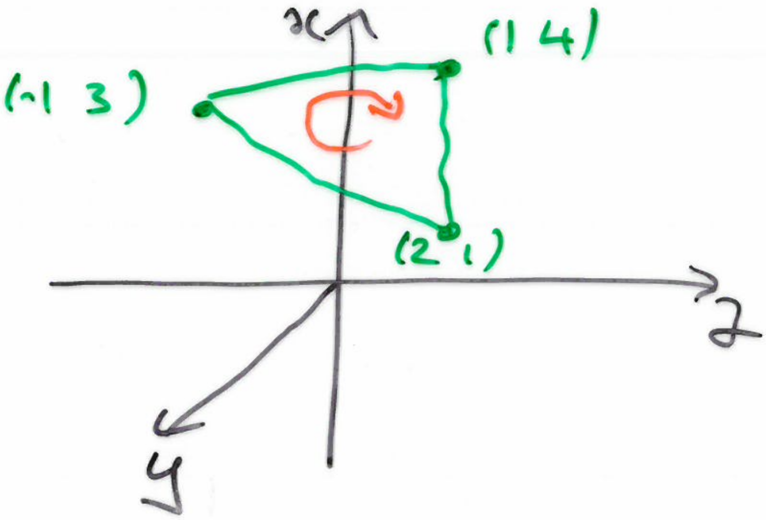
$$I = \int_S dy \wedge dz + \int_S dz \wedge dx + \int_S dx \wedge dy$$





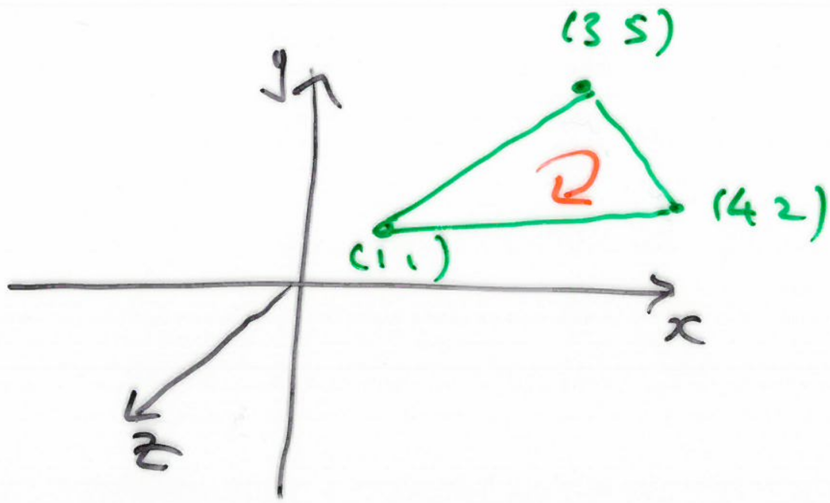
Area of triangle $= \frac{1}{2}\begin{vmatrix} 1 & 4 \\ -1 & -3 \end{vmatrix} = \frac{1}{2}$

$$\int_S dy \wedge dz = -\frac{1}{2}$$



$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} 3 & 2 \\ -2 & 1 \end{vmatrix} = \frac{7}{2}$$

$$\int_S dz \wedge dx = -\frac{7}{2}$$



$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = -5$$

$$\int_S dx \wedge dy = -5$$

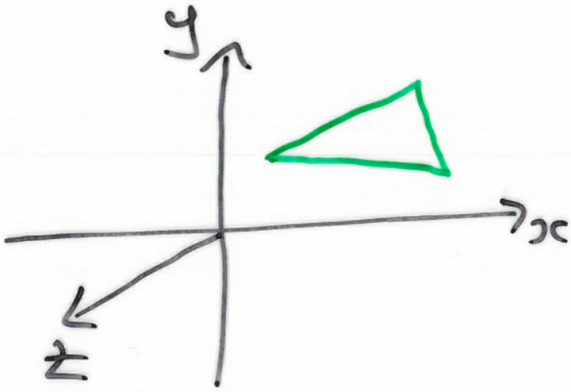
In conclusion:

$$\int_S dy \wedge dz + dz \wedge dx + dx \wedge dy$$

$$= -\frac{1}{2} - \frac{7}{2} - 5 = -9$$

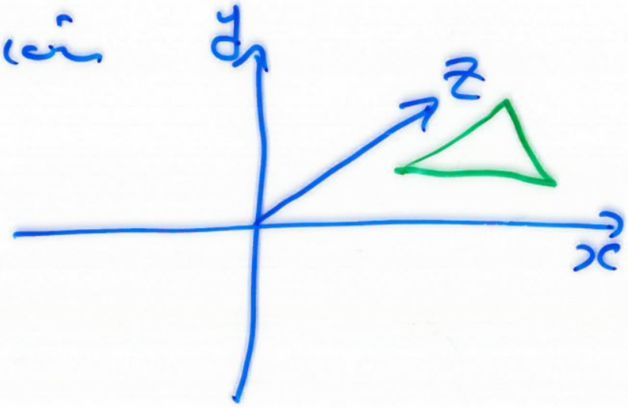
<u>Remark</u> $\int_S dx \wedge dy$ refers to

the orientation



$\int_S dy \wedge dx$ refers to the

orientation



So

$$\int_S dx \wedge dy = - \int_S dy \wedge dx$$

we write

$$dx \wedge dy = - dy \wedge dx$$

<u>Convention</u> $dx \wedge dy$, $dy \wedge dz$, $dz \wedge dx$ are the "orientations" when we view from the positive third axis.

So

$$\int_S dy \wedge dz = - \int dz \wedge dy$$

etc.

and $dy \wedge dz = - dz \wedge dy$

etc.