Summary of 1-forms and an introduction to 2-forms

- A 1-form is an expression such as

$$\omega = A\,dx + B\,dy$$

that can be integrated over oriented curves S.

- A 2-form is an expression such as

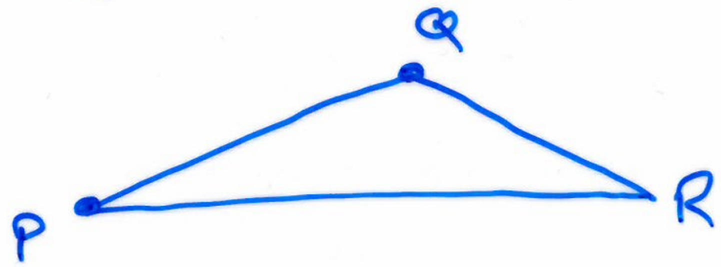
$$\omega = A\,dx \wedge dy + B\,dy \wedge dz + C\,dz \wedge dx$$

that can be "integrated" over "2-dimensional oriented regions"

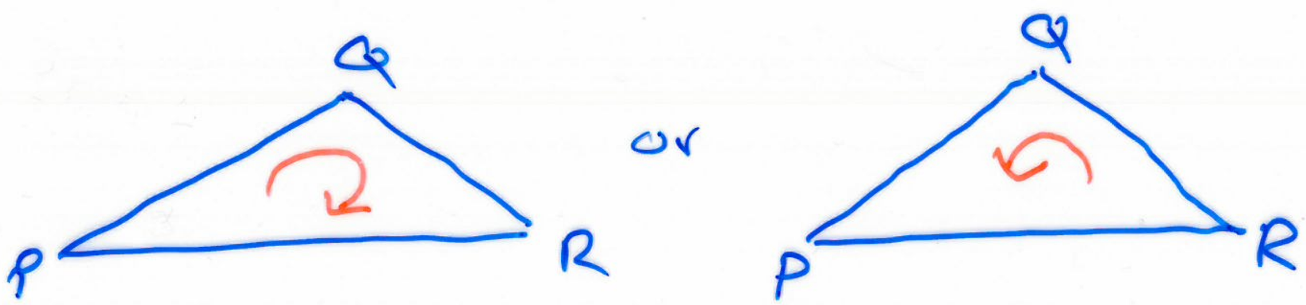
- Integrals of 1-forms are just limits of sums of integrals of constant 1-forms over oriented straight line segments.
- Integrals of 2-forms are just limits of sums of integrals of constant 2-forms over

Oriented planar triangles

Three points in a plane determine a triangle



An **orientation** of a triangle is specified by a curved arrow



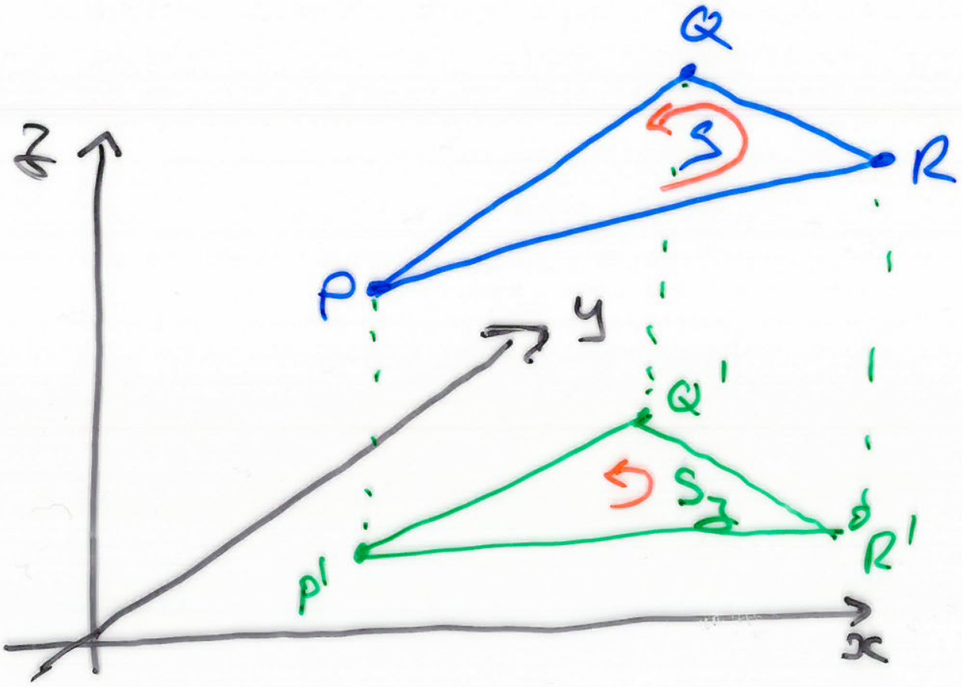
corresponding to one of two directions of rotation. The **positive side** of the triangle is the one from which the

arrow denotes anti-clockwise rotation.

An orientation is just an ordering of the vertices. The ordering QRP denotes the orientation on the above left triangle.

Constant 2-forms

Let S denote an oriented triangle in $\mathbb{R}^3$



Let S_z denote the image of the triangle S in the xy-plane under the projection

$$p_z : \mathbb{R}^3 \longrightarrow \mathbb{R}^2, \quad (x, y, z) \mapsto (x, y)$$

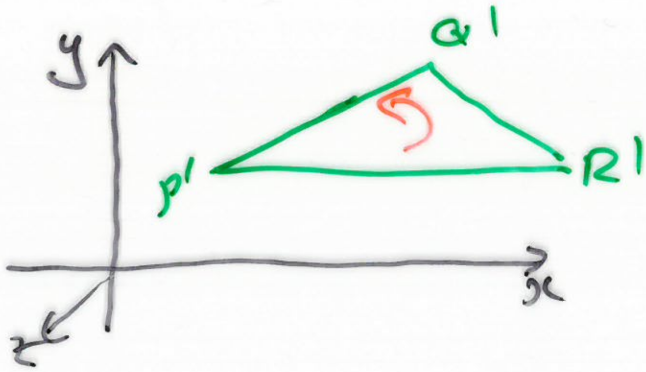
For any constant number $A \in \mathbb{R}$ let

$$\int_S A \, dx \wedge dy$$

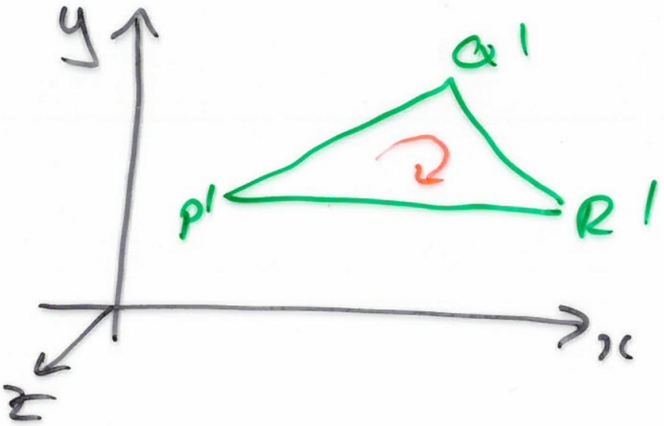
denote

$$\pm A \times (\text{area of } S_z)$$

with sign $+1$ if



and sign -1 if



Similarly define, for $B, C \in \mathbb{R}$,

$$\int_S B\, dz \wedge dx \quad \text{and} \quad \int_S C\, dy \wedge dz$$

Defn

$$\int_S A\, dx \wedge dy + B\, dy \wedge dz + C\, dz \wedge dx$$

$$= \int_S A\, dx \wedge dy + \int_S B\, dy \wedge dz + \int_S C\, dz \wedge dx$$

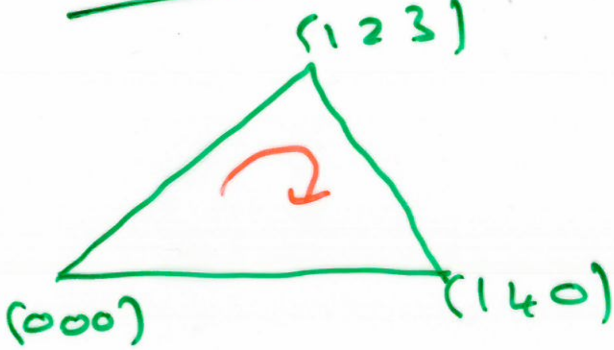
Example

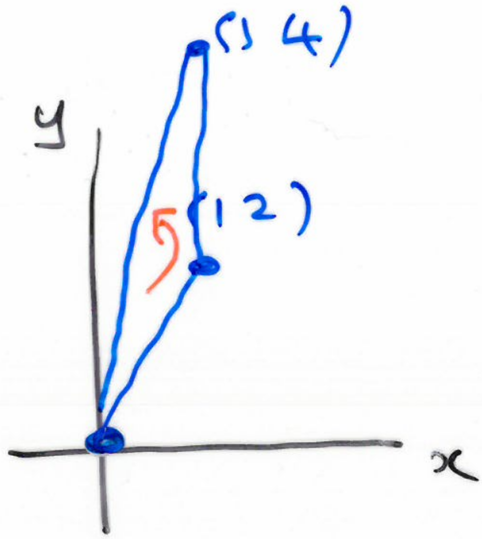
Evaluate

$$I = \int_S dx \wedge dy + 3\, dx \wedge dz$$

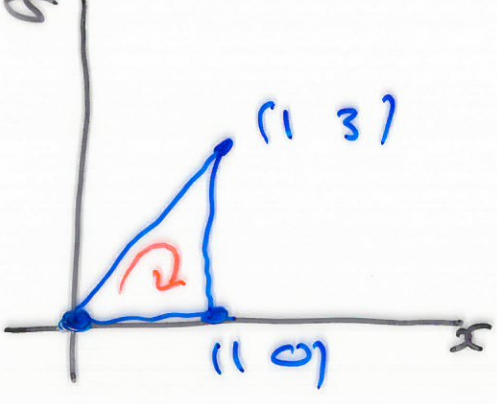
over the oriented triangle S with vertices (0,0,0), (1,2,3), (1,4,0) in that order.

Sol^n





Area of triangle = 1



Area $= \frac{1}{2} \cdot 3 = \frac{3}{2}$

so

$$I = \int_S dx \wedge dy + 3\, dx \wedge dz$$

$$= 1 - 3 \cdot \frac{3}{2} = -\frac{7}{2}$$