Example A particle is moved in a constant force field. It takes 3 units of work to move the particle from point (x, y) to point $(x+1, y)$. It takes 4 units of work to move the particle from point (x, y) to point $(x, y+1)$.

We say that work is represented by the 1-form

$$\omega = 3\,dx + 4\,dy$$

<u>Example</u> An investment portfolio involves two types of assets: type X and type Y. It costs €3 to acquire one unit of asset X, and -€3 to relinquish one unit of asset X. It costs €4 to acquire one unit of asset Y, and -€4 to relinquish one unit of Y.

We say that the marginal costs are represented by the 1-form

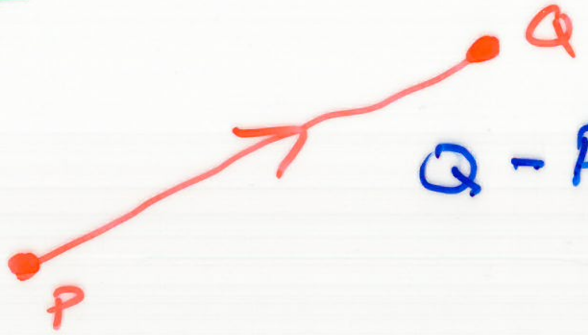
$$\omega = 3\,dx + 4\,dy.$$

Example Consider a particle in a constant force field, with work given by the 1-form

$$\omega = 2dx + 3dy + 5dz.$$

Calculate the work done in moving the particle along the straight line segment from point $P = (-1, 3, -5)$ to point $Q = (3, -1, 7)$.

Sol^n



$$Q - P = (4, -4, 12)$$

$$\text{work} = 2(4) + 3(-4) + 5(12)$$
$$= 56.$$

Example Find the 1-form

$\omega = A\,dx + B\,dy + C\,dz$

describing "work" in the constant force field where displacement of a particle from

$(0,0,0)$ to $(4,0,0)$ needs 3 units of work

$(1,-1,0)$ to $(1,1,0)$ " 2 " "

$(0,0,0)$ to $(3,0,2)$ " 5 " "

Sol^n

$$\left.\begin{aligned} 3 &= A.4 \\ 2 &= B.2 \\ 5 &= A.3 + C.2 \end{aligned}\right] \quad \begin{aligned} A &= \frac{3}{4} \\ B &= 1 \\ C &= \frac{11}{8} \end{aligned}$$

$5 = \frac{3}{4}.3 + 2C$

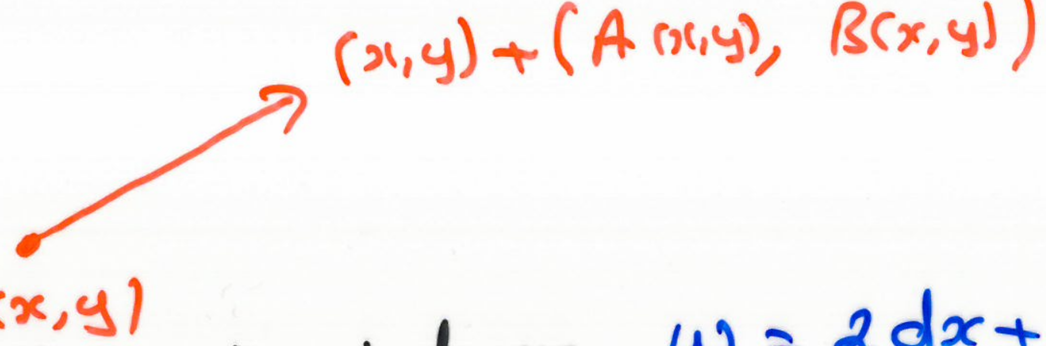
$5 = \frac{9}{4} + 2C \qquad \frac{20}{4} - \frac{9}{4} = 2C$

$\frac{11}{4} = 2C$

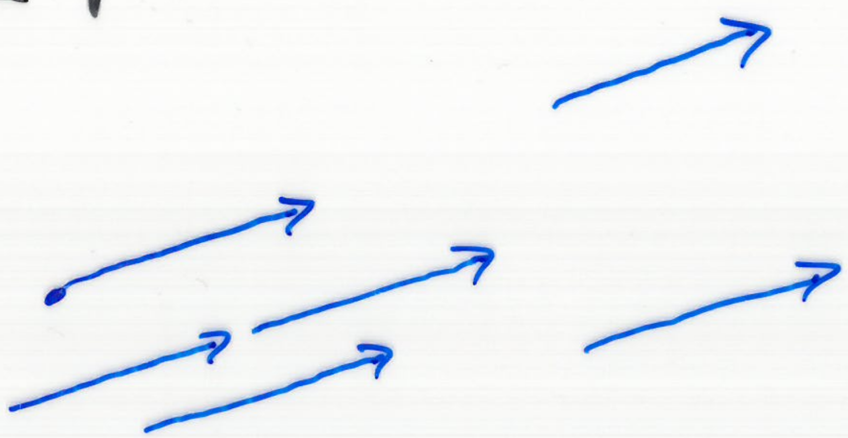
We can think of a 1-form

$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

as a collection of arrows in space (= the plane for two variables). For each point (x,y) in the plane we have the arrow



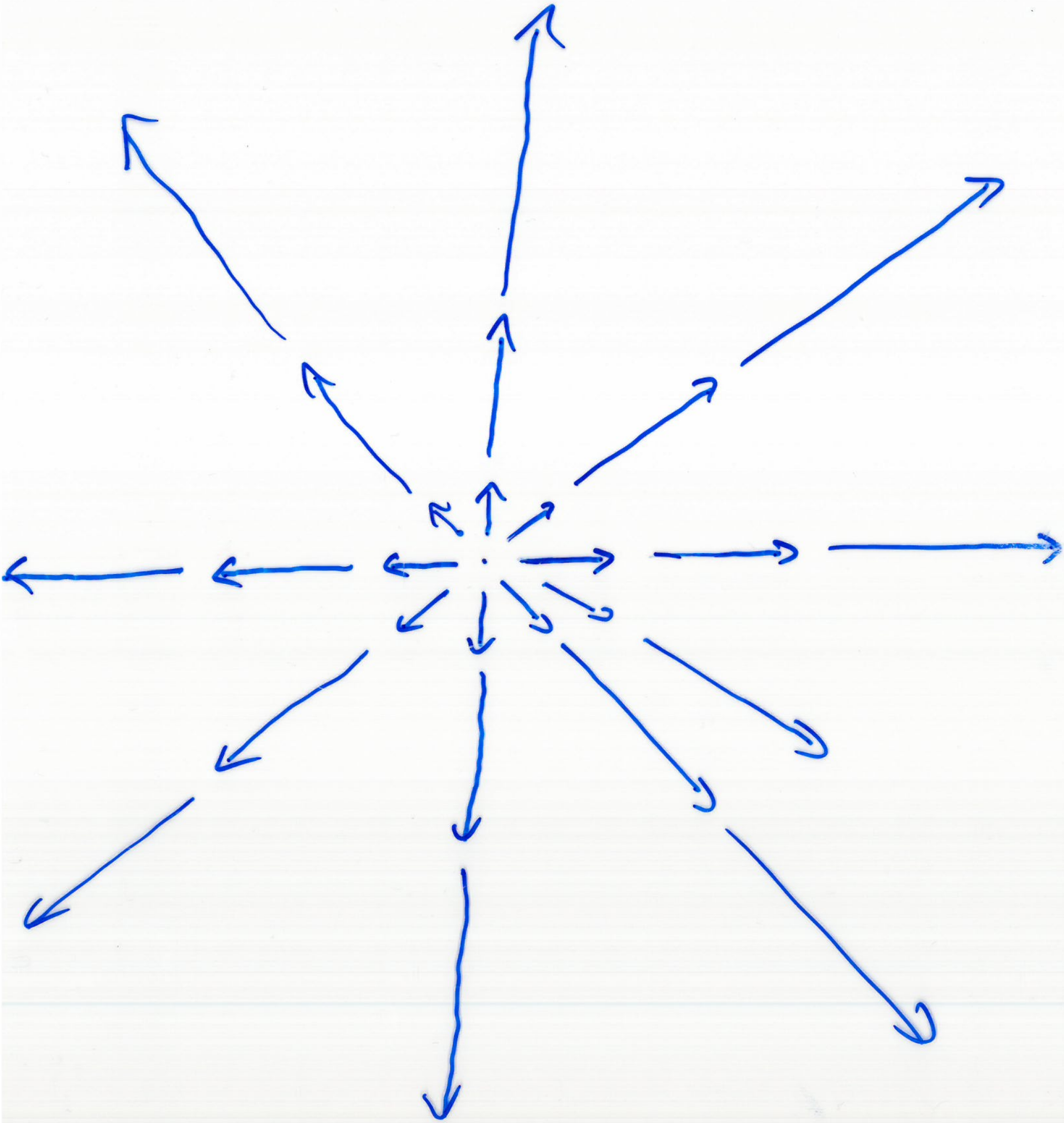
<u>Example</u> The 1-form $\omega = 2dx + dy$ can be pictured as



<u>Example</u> The 1-form

$$\omega = x\,dx + y\,dy$$

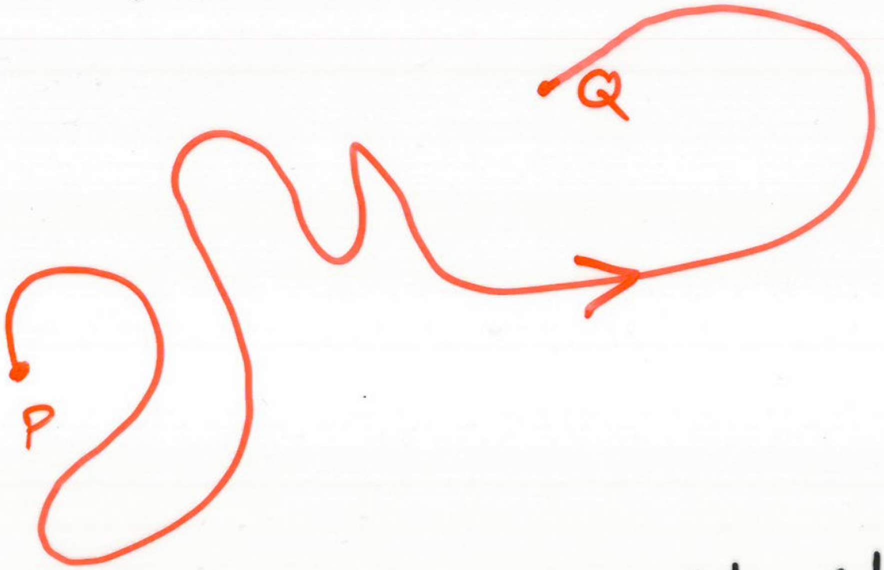
can be pictured as:



Integration of 1-forms

Let $\omega = A(x,y)\,dx + B(x,y)\,dy$ be a 1-form.

Let $S \subseteq \mathbb{R}^2$ be a 1-dimensional, oriented, connected subset.



Informally: If we think of ω as a "work 1-form" then

$$\int_S A(x,y)\,dx + B(x,y)\,dy$$

is the total work done in moving the particle from P to Q along S.