Tutorials : Thu 6 pm
Fri 12 noon

$$\int_{\partial S} \omega = \int_S d\omega \qquad *$$

For one variable ($n=1$) and 0-forms ω ($p=0$) we understand all terms in the formula except for $d\omega$.

<u>Definition</u> For a differential 0-form $\omega = F(x)$ we define the 1-form

$$d\omega = F'(x)\,dx$$

we call $d\omega$ the <u>total derivative</u> of ω, or just the <u>derivative</u> of ω.

So (*) for $n=1$, $p=0$ is just the Fundamental Theorem of Calculus.

Let's recall from 1st year

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum f(v_i)(x_i - x_{i-1})$$

where

- $P = \{a = x_0, x_1, x_2, \dots, x_n = b\}$ is a sequence of points in $[a,b]$.
- $v_i \in [x_{i-1}, x_i]$
- $\|P\| = \max_{1 \le i \le n} |x_i - x_{i-1}|$

Proof of the fundamental theorem of Calculus

Suppose that the Galway to Dublin train has a functioning speedometer, but a broken mileometer. The driver has a clock.

To estimate the distance travelled from time $t = a$ to

time $t = b$ the driver could calculate

$$\sum_{i=1}^{n} f(t_i)(t_i - t_{i-1})$$

where $f(t)$ is the speed of the train at time t, and

$a = t_0 < t_1 < t_2 < \dots < t_n = b$

Let

$F(t) =$ the total distance travelled at time t.

now

$$f(t) = F'(t)$$

and roughly

$$F(b) - F(a) \simeq \sum_{i=1}^{n} f(t_i)(t_i - t_{i-1}).$$

Taking limits as $\|P\| \to 0$

$$F(b) - F(a) = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(t_i)(t_i - t_{i-1})$$

Thus

$$\boxed{F(b) - F(a) = \int_a^b f(t)\, dt}$$

Fundamental Theorem of Calculus

or, for $\omega = F(t)$ and $S = [a, b]$

$$\boxed{\int_{\partial S} \omega = \int_S d\omega}$$

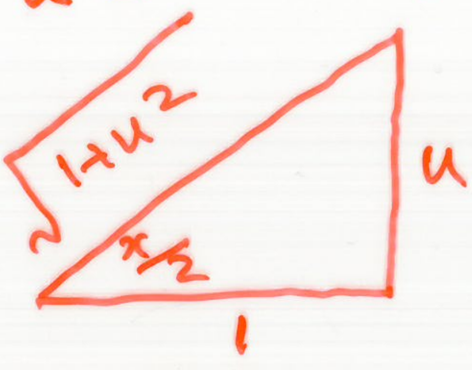
Example Find a differential 0-form ω whose total derivative $d\omega$ is

$$d\omega = \frac{1}{5+3\cos(x)}\, dx$$

Sol^n using language of 1st year maths, we want to find

$$\omega = \int \frac{1}{5+3\cos(x)}\, dx$$

Let $u = \tan\left(\frac{x}{2}\right)$



$$\sin\left(\frac{x}{2}\right) = \frac{u}{\sqrt{1+u^2}}, \quad \cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+u^2}}$$

$$du = \frac{1}{2} \sec^2\left(\frac{x}{2}\right) dx$$

now

$w = \ldots$ etc.