Example Integrate the 0-form $\omega = 3x^2 + 4$ on ∂S where $S = [2,1] \cup [3,4]$.

Soln

$$\int_{\partial S} \omega \underset{\uparrow \text{definition}}{=} \int_{\partial [2,1]} \omega + \int_{\partial [3,4]} \omega$$

$$= 3(1^2) + 4 - 3(2^2) - 4 + 3(4^2) + 4 - 3(3^2) - 4$$

$$= \cancel{7 - 12 - 4 + 27 + 4 - 48 - 4}$$

$$= -9 + 43 - 27$$

$$= 12.$$

Text: Advanced Calculus
by M. Spiegel
(Schaum's Series)

Background reading:
"Advanced Calculus: a differential forms approach" by
Harold M. Edwards

Also: Spivak's books on manifolds.

Continuous assessment (30%)
Three in-class tests (each 10%)
The tests will be taken from the more-or-less verbatim from the homework sheet.
Final exam: 70%
Lecture Notes & problem sheet on Blackboard.
Tutorials: Tue & Thu 6pm

Stokes Formula

$$\int_{\partial S} \omega \checkmark = \int_{S} d\omega \,?$$

n variables, $n = 1$.

$p = 0$, ω is a 0-form

Differential 1-forms in 1 variable

A differential 1-form is a function of the form

$$\omega = f(x)\,h$$

which inputs numbers $x, h \in \mathbb{R}$ and returns the number $f(x)\,h$, where $f(x)$ is some function.

Example Evaluate the 1-form

$$\omega = (x^2+6)\,h$$

at $x=2$, $h=0.5$.

Soln $(2^2+6)\,0.5 = 5$

Notation: We usually denote the 1-form

$$\omega = f(x)\,h$$

by

$$\omega = f(x)\,dx$$

Example Evaluate the 1form

$$\omega = \sin(x)\,dx$$

at $x = \frac{\pi}{2}$, $dx = 0.25$

Soln

$$\sin\left(\frac{\pi}{2}\right) \times 0.25 = 0.25.$$

Defn Given a 1-form

$$\omega = f(x)\,dx$$

and an oriented interval $S = [a, b]$ we define the integral as

$$\int_S \omega = \int_a^b f(x)\,dx$$

explained in 1st year.

Informally: $\int_a^b f(x)\,dx$ is the area between the curve $y = f(x)$ and the x-axis from a to b, where if $b > a$ areas above the x-axis are regarded

as positive, and areas below the x-axis are regarded as negative.

Problem A fundraising project has daily expenditure of $10 000. The rate of contributions at time t is modelled by

$$c(t) = -100t^2 + 20\,000.$$

What net proceeds can be expected?

Soln Project runs until

$$c(t) \leq 10\,000.$$

$$-100t^2 + 20\,000 = 10\,000$$

$$100t^2 = 10\,000$$

$$t = 10.$$

The project will run from $t=0$ to $t=10$.

Contributions are modelled by the 1-form

$$u = (-100t^2 + 20\,000)\,dt$$

Expenditure is modelled by the 1-form

$$v = -10\,000\,dt$$

The net rate of income is modelled by the 1-form

$$w = u + v = (100t^2 + 10\,000)\,dt$$

The project can be expected to make

$$\int_S w \,.$$

where $S = [0, 10]$

$$\int_S \omega \overset{\text{definition}}{=} \int_0^{10} -100t^2 + 10\,000\,dt$$

$$= \frac{-100t^3}{3} + 10\,000t \Big|_0^{10}$$

$$= \$66\,666.67$$