A 1-form $\omega = A\,dx + B\,dy + C\,dz$ is something that we can integrate over a 1-dimensional region $S$.

1-forms represent abstract notions such as force fields, marginal costs. But their integral $\int_S \omega$ is something that is easy to understand: work, total cost etc.

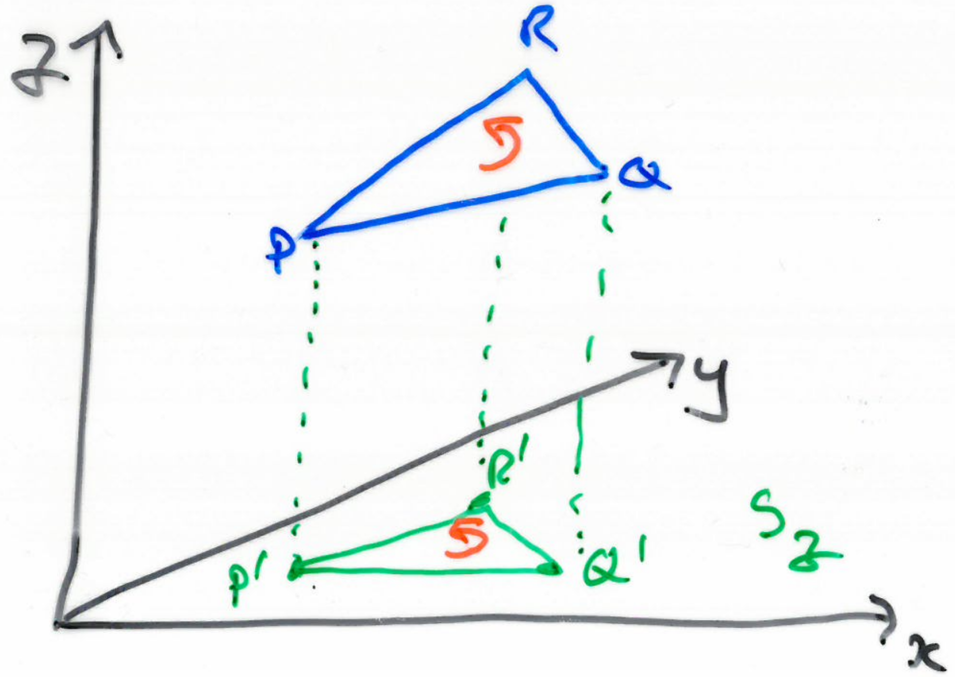
A 2-form

$$\omega = A\,dx \wedge dy + B\,dy \wedge dz + C\,dz \wedge dx$$

is something that we can integrate over a 2 dimensional region.

# Constant 2-forms

Let $S$ denote an oriented triangle in $\mathbb{R}^3$




Let $S_z$ denote the image of $S$ in the $xy$-plane under the projection

$$p_z : \mathbb{R}^3 \to \mathbb{R}^2, \quad (x,y,z) \longmapsto (x,y)$$

For any constant $A \in \mathbb{R}$

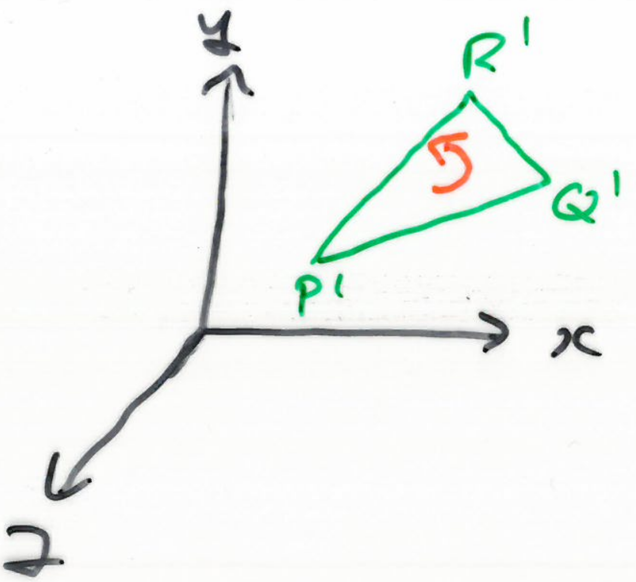
let

$$\int_S A \, dx \wedge dy$$

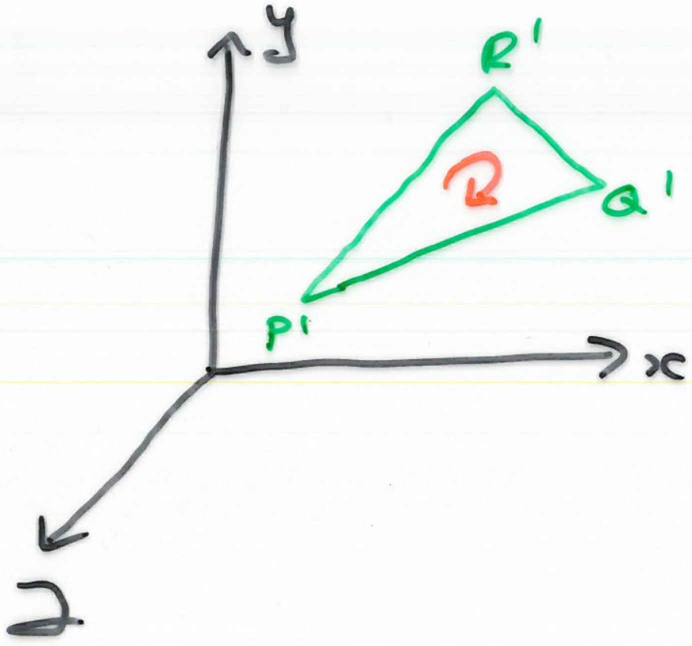
denote

$$\pm A \times (\text{area of } S_2)$$

with sign $+1$ if




and with sign $-1$ if




Similarly define for $B, C \in \mathbb{R}$

$$\int_S C \, dz \wedge dx$$

and

$$\int_S B \, dy \wedge dz$$

**Defn**

$$\int_S A \, dx \wedge dy + B \, dy \wedge dz + C \, dz \wedge dx$$

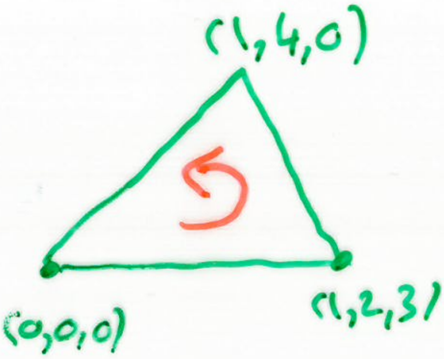
$$= \int_S A \, dx \wedge dy + \int_S B \, dy \wedge dz + \int_S C \, dz \wedge dx$$

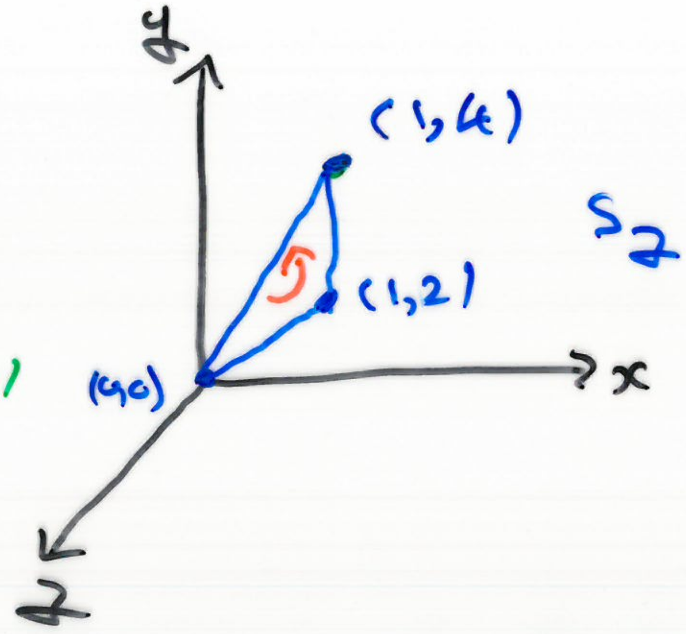
**Example** Evaluate

$$I = \int_S dx \wedge dy + 3\, dz \wedge dx$$

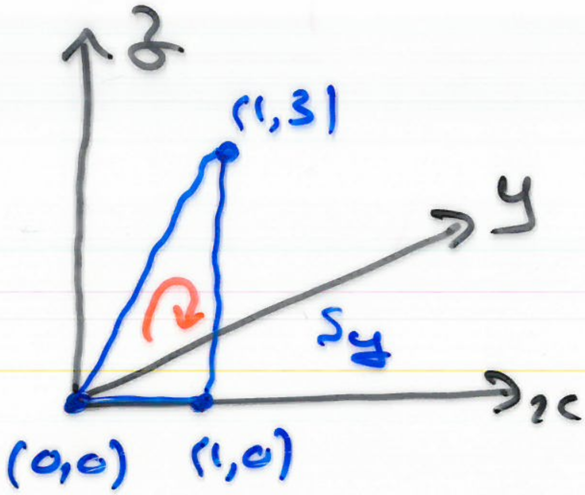
over the oriented triangle $S$ with vertices $(0,0,0)$, $(1,2,3)$ and $(1,4,0)$ in that order.

**Sol$^n$** $I = \int_S dx \wedge dy + \int_S 3\, dz \wedge dx$




area of $S_z$ = 1




area of $S_y = \frac{3}{2}$

So

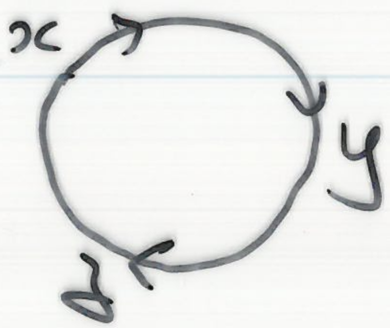
$$I = \int_S dx \wedge dy + \int_S 3\, dz \wedge dx$$

$$= +1.1 + (+)\,3.\frac{3}{2}$$

$$= \frac{11}{2}.$$

---

Always use $dx \wedge dy$, $dy \wedge dz$, $dz \wedge dx$

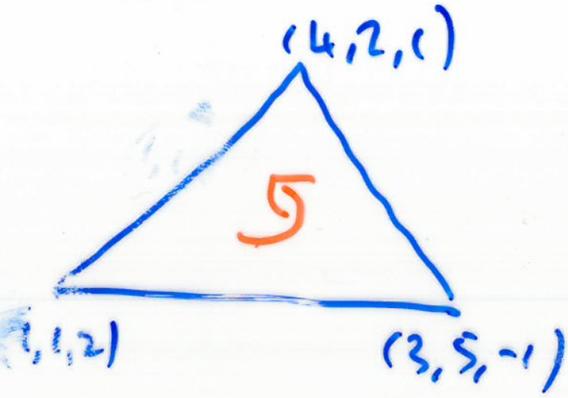


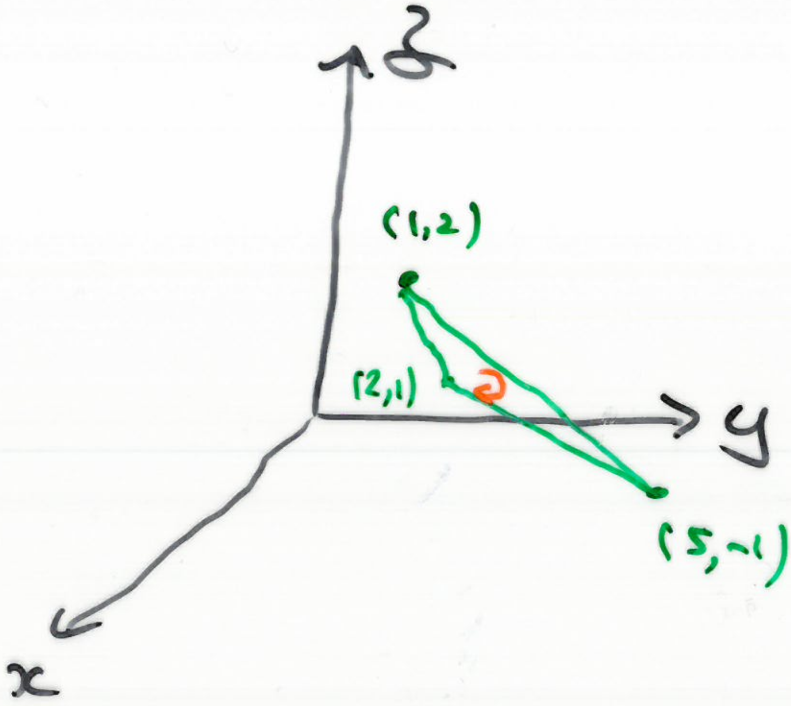
**Example** Evaluate

$$I = \int_S dy \wedge dz$$

where $S$ is the oriented triangle with vertices $(1,1,2)$, $(3,5,-1)$, $(4,2,1)$ in that order.

**Sol$^n$**




$$\text{area of triangle} = \frac{1}{2}\begin{vmatrix} 1 & 4 \\ -1 & -3 \end{vmatrix} = \frac{1}{2}$$

$$I = \int_S dy \wedge dz = -\frac{1}{2}$$