MA2286 Differential Forms

(= Calculus)

Aim: Explain and apply the generalized Stokes formula:

$$\int_{\partial S} \omega = \int_S \partial\omega$$

where

- ω is a differential p-form in n variables.
- S is a nice region in $\mathbb{R}^n$.
- ∂S is the boundary of the region
- $\int$ is ~~the~~ an integral

Differential 0-forms in 1 variable

($p=0$, $n=1$)

A differential 0-form in 1-variable is just a differentiable, real valued function

$$\omega = f(x)$$

Examples $\omega = 3x-4$

$\omega = 3x^2+4$

$\omega = \sin(x)$

are differential 0-forms.

Usually a diff. 0-form is given in the context of some closed interval

$$S = [a, b] \subseteq \mathbb{R}$$

or a union of closed intervals

$$S = [a_1, b_1] \cup [a_2, b_2] \cup \dots \cup [a_k, b_k].$$

we only require ω to be differentiable at points in S. 3

Example

$$\omega = |x|$$

is a differential 0-form on

$$S = [1, 100]$$

clearly ω is not a diff. 0-form on $[-10, 1]$

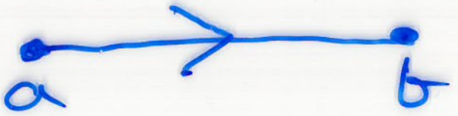
Terminology

I'll say <u>0-form</u> instead of differential 0-form.

For $a < b \in \mathbb{R}$ we write

$$[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and we picture this as

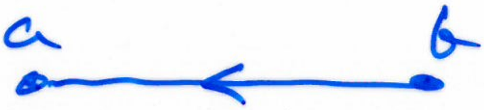


The arrow is an orientation that specifies the direction of travel from a to b.

For $a < b$ we write

$$[b, a] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and picture this as



We say that $[a, b]$ and $[b, a]$ are oriented intervals.

Example $S = [2,1] \cup [3,4] \cup [6,5]$

The boundary of the oriented interval $S = [a, b]$ is the set

$$\partial S = \{a, b\}$$

is the set consisting of two points, the initial point a and the final point b.

<u>Example</u> $S = [2,1] \cup [3,4] \cup [6,5]$

$$\partial S = \{2, 1, 3, 4, 6, 5\}$$

<u>Terminology</u> we'll say that $S = [a, b]$ is <u>1-dimensional</u>, and that ∂S is <u>0-dimensional</u>

Definition Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = F(b) - F(a)$$

final point   initial point

Example Integrate the differential 0-form $\omega = 3x^2 + 4$ over the boundary of the oriented interval $S = [2, 1]$.

Sol^n

$$\int_{\partial S} \omega = \omega(1) - \omega(2)$$

$$= 7 - 16 = -9$$