Second homework now online

Intermediate Value Theorem

Suppose

$$y = f(x)$$

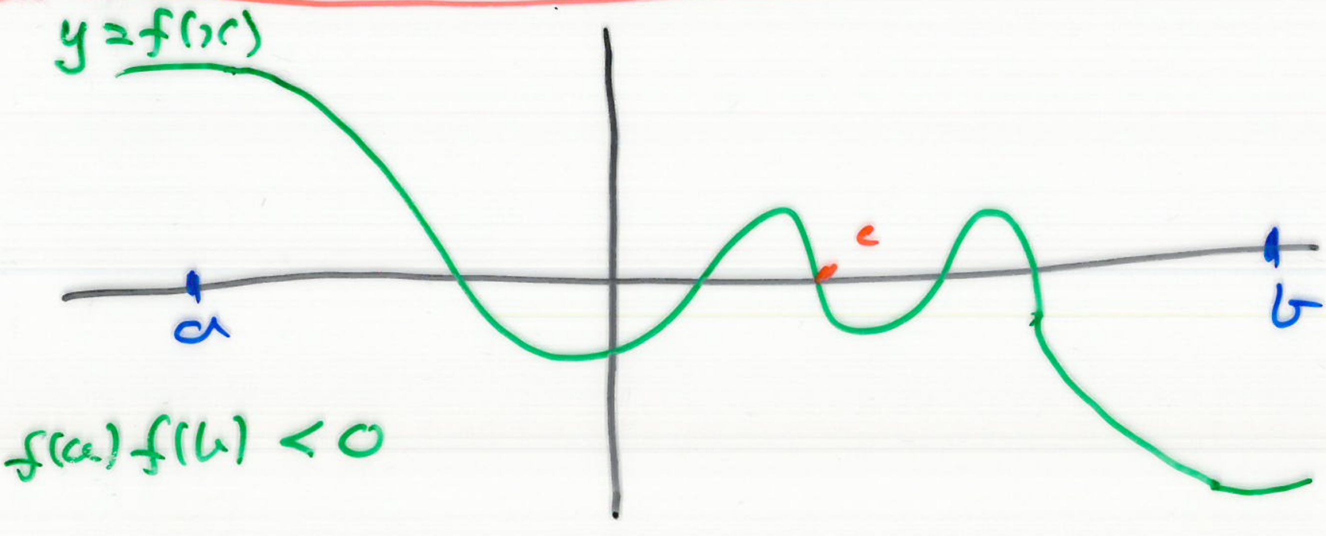
is continuous at all points x in a range $a \le x \le b$.

Suppose also that

$$f(a)\,f(b) \le 0 .$$

Then there exists at least one value c in the range $a \le c \le b$ such that

$f(c) = 0$.



Example Show that

$$x^3 - x - 1 = 0$$

has a solution in the range

$1 \le x \le 2$.

Soln Set $f(x) = x^3 - x - 1$.

"clearly" this is continuous.

$a = 1, \quad b = 2$.

$$\left.\begin{array}{l} f(1) < 0 \\ f(2) > 0. \end{array}\right\} \quad f(1)f(2) < 0.$$

The IVT says that there is at least one value c, $a \le c \le b$, such that

$$f(c) = 0.$$

i.e. c is a solution to the equation $x^3 - x - 1 = 0$.

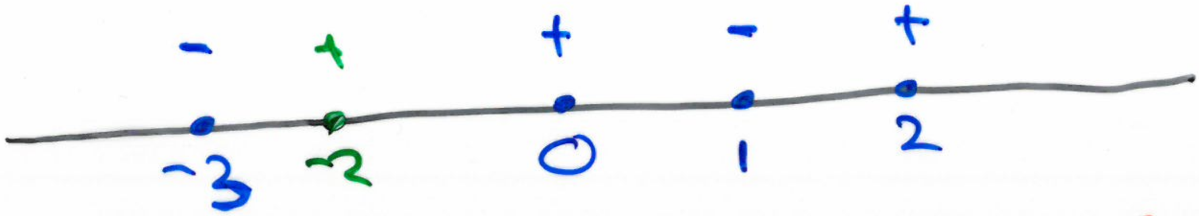
Example Show that

$$x^3 - 4x + 1 = 0$$

has three real solutions, and find approximations to them.

Sol^n

$$f(x) = x^3 - 4x + 1$$



$f(0) > 0$

$f(1) < 0$

$f(2) > 0$

$f(-3) < 0$

$f(-2) > 0$

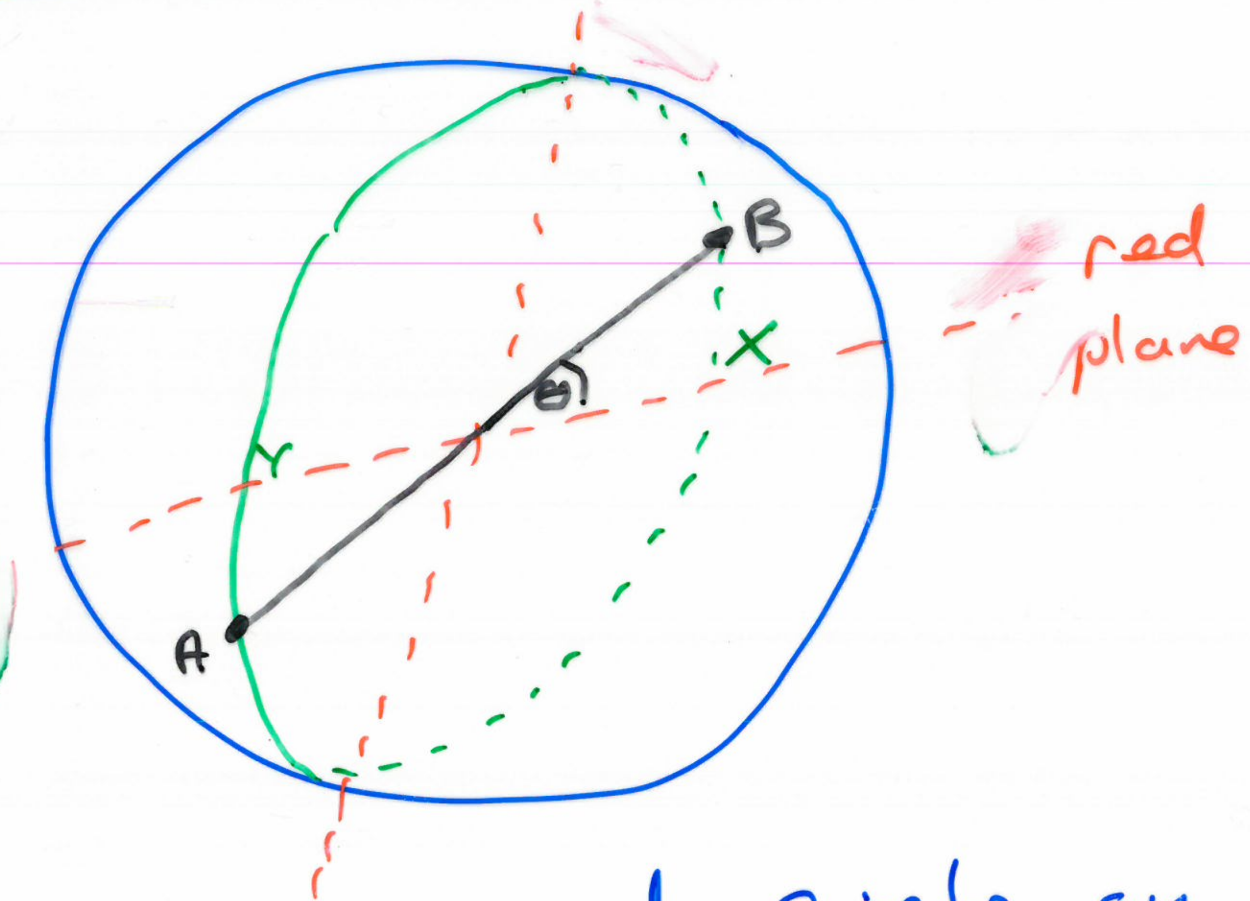
IVT says that $f(x) = 0$ has three solutions:

$c_1 \in [-3, -2]$

$c_2 \in [0, 1]$

$c_3 \in [1, 2]$.

Application of IVT



Take any great circle on the Earth.

__Fact__: There exist two opposite points on your great circle with equal air pressure.

__Explanation of this fact__

Consider

$f(\theta) =$ air pressure at A

$\quad -$ air pressure at B.

Note: $f(\theta)$ is a continuous function of θ.

I want to prove that for some value of the angle θ pressure at A = pressure at B.

i.e. I want to prove that

$f(\theta) = 0$

for some angle $\theta \in [0, \pi]$.

If $f(0)=0$, or if $f(\pi)=0$ then we are done!

Suppose then that

$f(0) \neq 0$ and $f(\pi) \neq 0$.

Note: $f(0) f(\pi) < 0$

$f(0) =$ air pressure at X

$-$ air pressure at Y

$f(\pi) =$ air pressure a Y

$-$ air pressure at X.

So $f(0) = -f(\pi)$.

So IVT says there is some $\theta \in [0, \pi]$ such that $f(\theta) = 0$.

QED