

## Functions

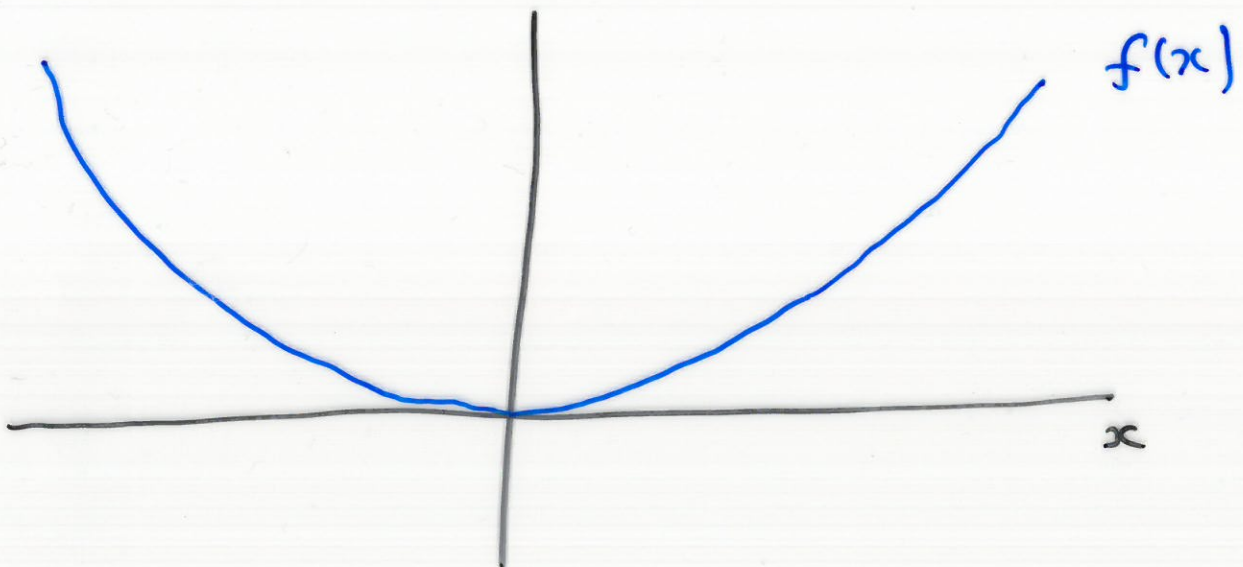
A function  $f: D \rightarrow C$  consists of

- 1) a set  $D$  called the domain.
- 2) a set  $C$  called the codomain.
- 3) a rule that assigns precisely one element  $f(x) \in C$  to each  $x \in D$ .

Example  $f: \mathbb{R} \rightarrow \mathbb{R}$  with  $f(x) = x^2$ .

$$D = \mathbb{R}, C = \mathbb{R}.$$

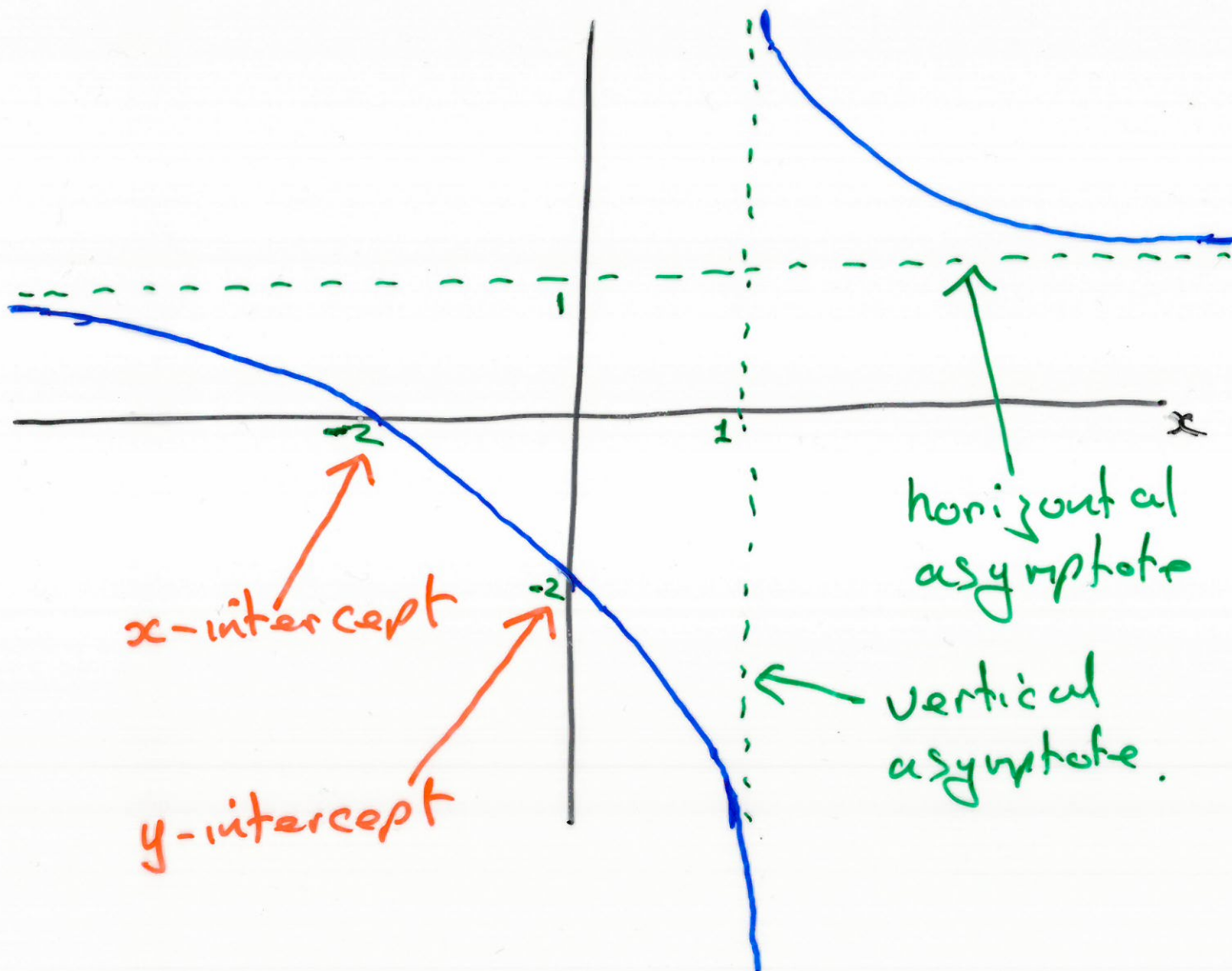
its graph is:



Example  $g: \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R}$

with  $g(x) = \frac{x+2}{x-1}$ .

$D = \mathbb{R} \setminus \{1\}$ ,  $C = \mathbb{R}$ .



## Convention

often we use a formula such as  $h(x) = \frac{x+2}{x-3}$  to describe

a function, without explicitly stating what the domain and codomain are.

In this situation we always assume that the domain is the largest possible subset of  $\mathbb{R}$  for which the formula makes sense. we take the codomain to be  $\mathbb{R}$ .



## Examples

$$1) \quad h(x) = \frac{x+2}{x-3}$$

$$D = \mathbb{R} \setminus \{3\}$$

$$C = \mathbb{R}$$

$$2) \quad g(x) = \sqrt{x} = \text{positive square root of } x.$$

$$D = \{x \in \mathbb{R} : x \geq 0\}$$

$$= [0, \infty)$$

$$3) \quad p(x) = \sqrt{x^2 - 1}$$

$$D = \{x \in \mathbb{R} : x \leq -1\} \cup \{x \in \mathbb{R} : x \geq 1\}$$

$$= (-\infty, -1] \cup [1, \infty)$$

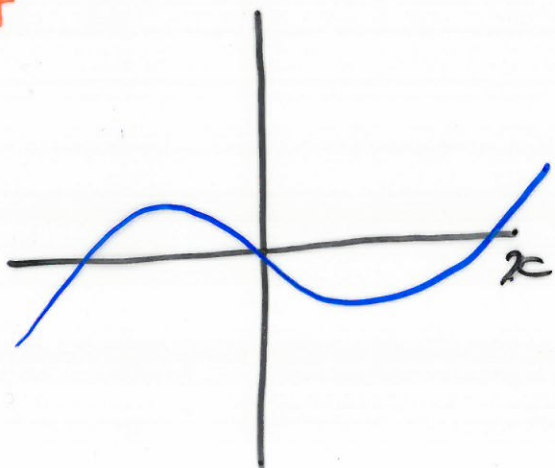
$$4) \quad r(x) = \sqrt{\frac{1}{x}}$$

$$D = \{x \in \mathbb{R} : x > 0\}$$

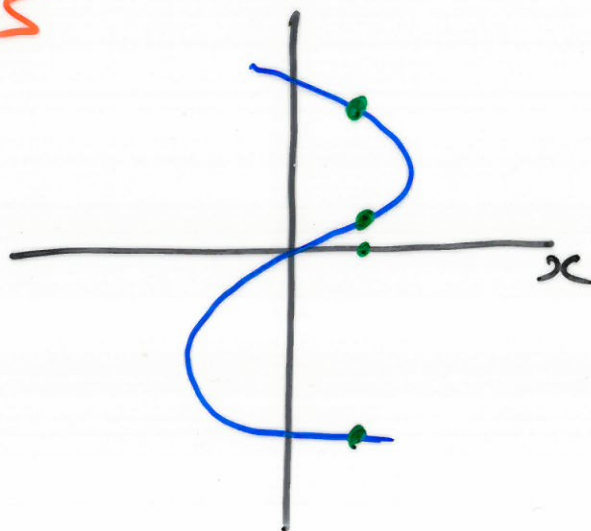
$$= (0, \infty)$$

Question: Which of the following are graphs of functions of  $x$ ?

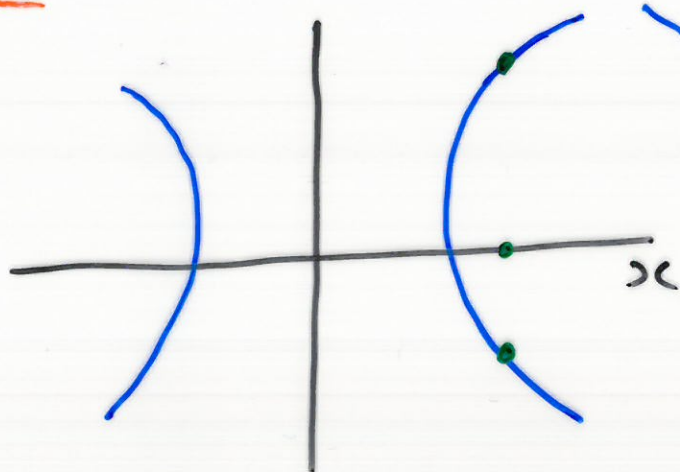
A



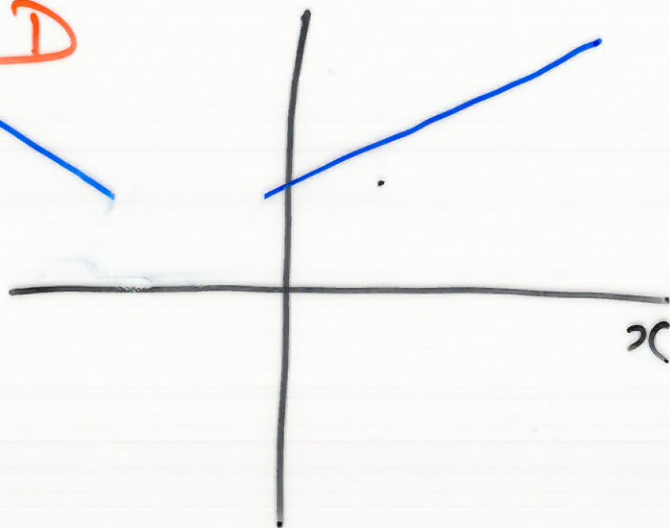
B



C



D



B not a function  
C not a function  
A is a function  
D is a function