The Real Stuff!

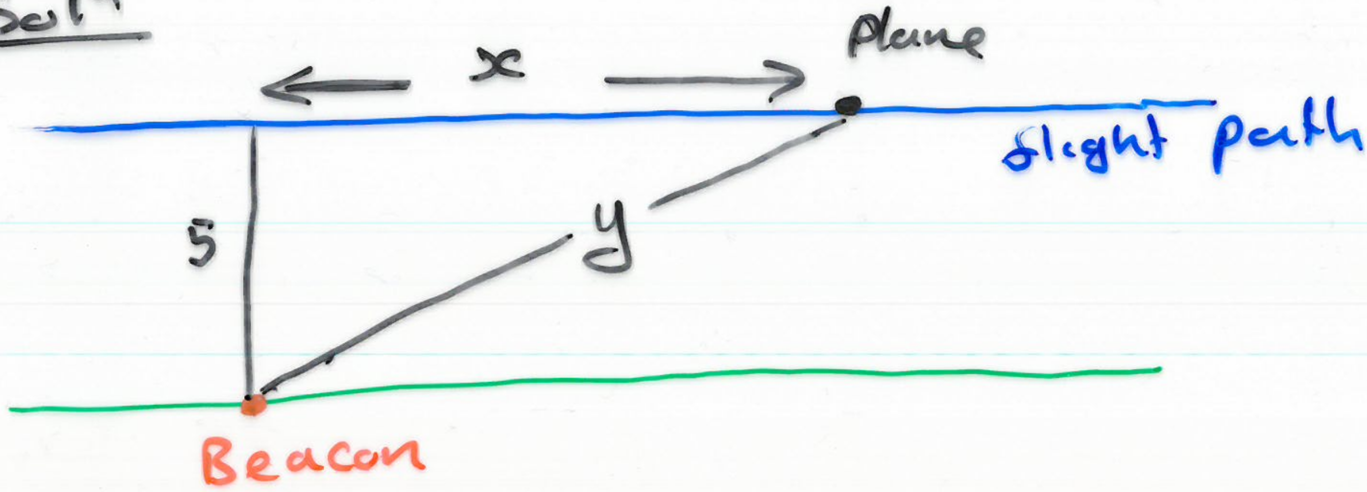
Applications

The derivative can be thought of as a rate of change.

Problem

An aircraft is flying horizontally at a speed of 600 km/h. How fast is the distance between the aircraft and a radio beacon increasing 1 minute after the plane passes 5 km directly above the beacon?

Sol^n



want to find

$$\boxed{\frac{dy}{dt} \text{ when } t = 1}$$

when $t = 1$, $x = 10$

$$\boxed{5^2 + x^2 = y^2} \quad (*)$$

for all t.

Chain rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Differentiate both sides of (*) with respect to t.

$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt} \qquad (**)$$

$\frac{dx}{dt} = 10$

So when $t=1$ equation (**) becomes

$$2.10.10 = 2\sqrt{5^2+10^2}\ \frac{dy}{dt}$$

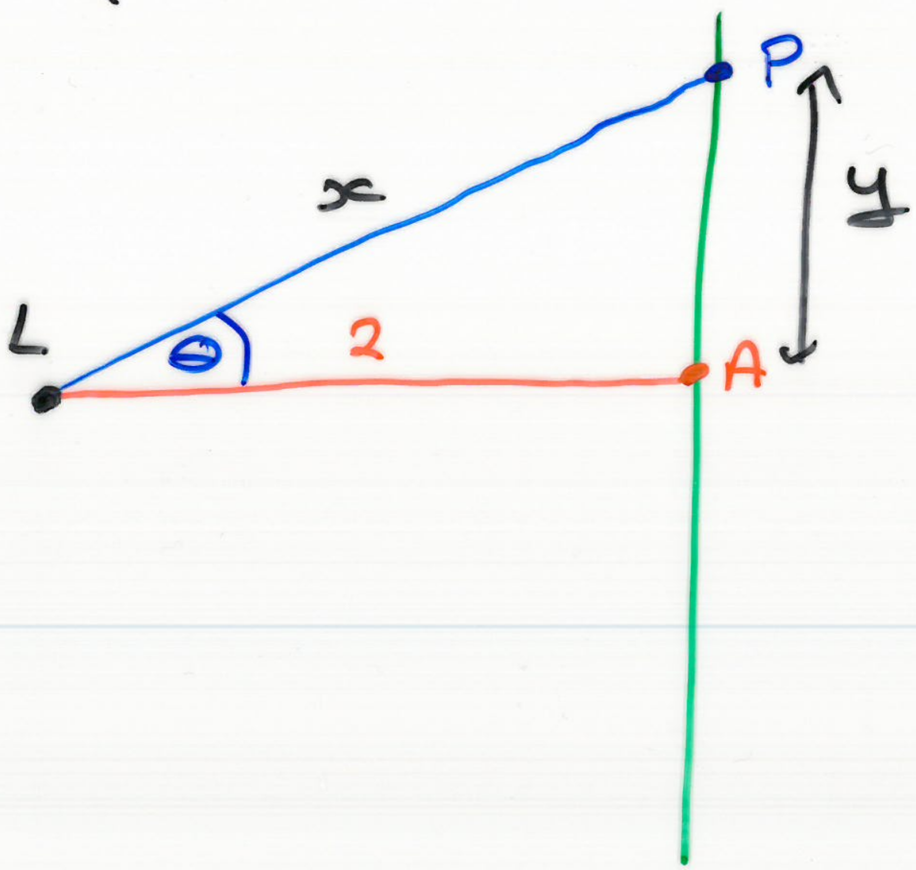
$$\frac{200}{2\sqrt{125}} = \frac{dy}{dt}$$

$$\frac{100}{\sqrt{5.25}} = \frac{dy}{dt}$$

$$\frac{100}{5.\sqrt{5}} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{20}{\sqrt{5}} \text{ km/min}$$

<u>Problem</u> A lighthouse L is located on a small island 2 km from the nearest point A on a long straight shoreline. The lighthouse light rotates at 3 revs per minute. How fast is the illuminated spot P on the shoreline moving when it is 4 km from A?



We need to find

$$\boxed{\frac{dy}{dt} \quad \text{when} \quad y = 4}$$

$$\frac{d\theta}{dt} = 6\pi \qquad \text{radians/min}$$

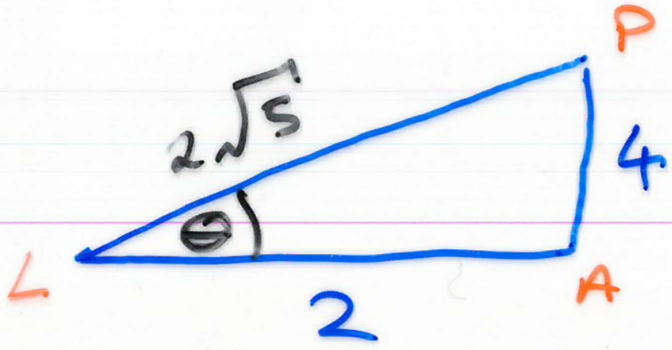
Both θ and y are functions of t.

$$\tan\theta = \frac{y}{2} \qquad (*)$$

Differentiate both sides of $(*)$ with respect to t.

$$\boxed{\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt} \qquad (**)}$$

when $y = 4$



$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{2\sqrt{5}}{2}.$$

$$= \sqrt{5}$$

From $(**)$, when $\underline{\underline{y = 4}}$

$$\frac{dy}{dt} = 2\sec^2\theta \frac{d\theta}{dt}$$

$$= 2 . 5 . 6\pi \quad \text{km/min}$$

$$= 60\pi \quad \text{km/min}.$$