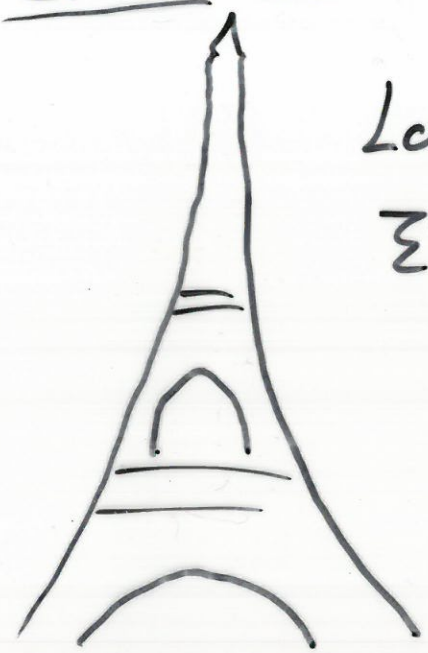
Calculus

Topics

1) Functions & Limits
2) Rates of change
3) Differential Equations

Text: Stewart, "Calculus"

Quick Introduction to (1) & (2).



La Tour Eiffel

↕ Stone falls y metres in t secs

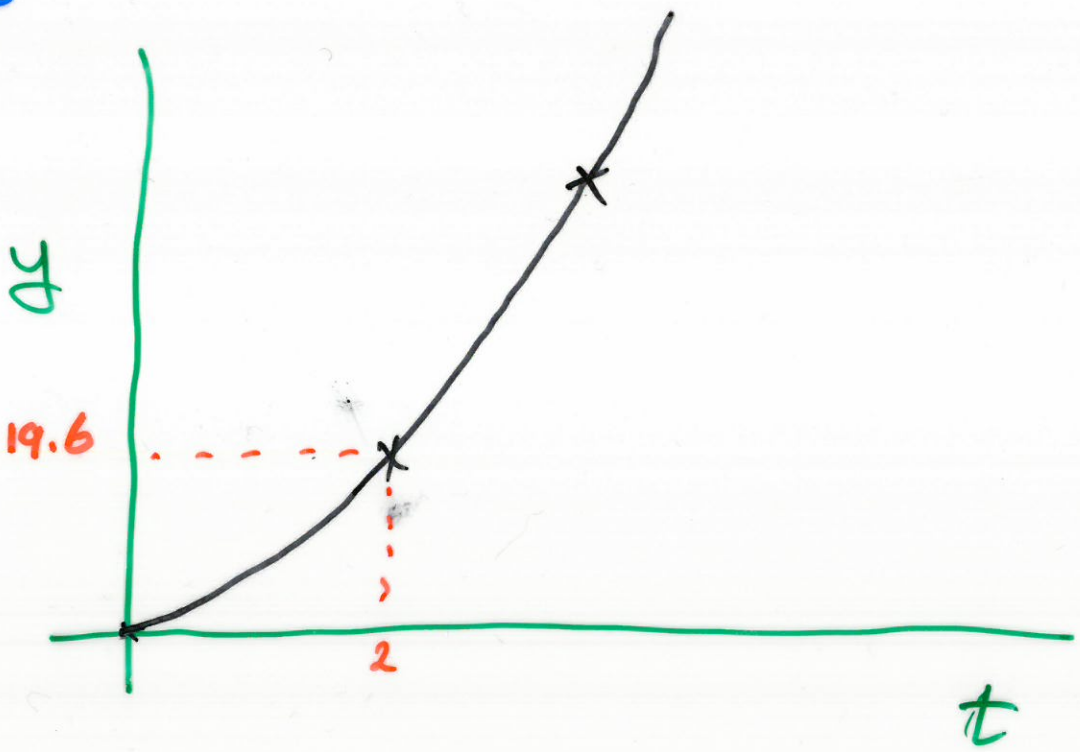
Experiment suggests

$$y = 4.9t^2$$

We say that **y is a function of t.** This means:

for each value of t there is **one** corresponding value of y.

Functions are represented by their graph



<u>Question</u> What is the average speed of the stone between $t=2$ and $t=3$ secs?

Soln

Average speed = $\frac{\text{distance travelled}}{\text{time}}$

$$= \frac{y(3) - y(2)}{1}$$

$$= \frac{4.9(9-4)}{1} = 4.9 \times 5$$

$$= 24.5 \text{ m/sec}.$$

Question what is the speed at time $t = 2$?

The speed at $t = 2$ is

$$v(2) = \lim_{h \to 0} \frac{y(2+h) - y(2)}{h}$$

(4)

$$= \lim_{h \to 0} \frac{4.9(2+h)^2 - 4.9(2^2)}{h}$$

$$= \lim_{h \to 0} 4.9 \left(\frac{4 + 4h + h^2 - 4}{h} \right)$$

$$= \lim_{h \to 0} 4.9 \left(\frac{h(4+h)}{h} \right)$$

$$= \lim_{h \to 0} 4.9 (4 + h)$$

$$= 19.6 \quad m/sec .$$