

Matrix Multiplication

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So $AB = I$

So $A^{-1} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$

1) Where did he get B ?

2) who cares & why?

1) Gauss-Jordan method for finding the inverse of A.

$$(A \mid I) \xrightarrow[\text{operations}]{\text{row}} (I \mid B)$$

Then $B = A^{-1}$.

There are three types of row operation:

(I) $R_i \mapsto R_i + \lambda R_j \quad (i \neq j)$

(II) $R_i \longleftrightarrow R_j$

(III) $R_i \rightarrow \lambda R_i \quad (\lambda \neq 0)$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_2 \mapsto R_2 - 2R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_3 \mapsto R_3 - 3R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -3 & 0 & 1 \end{array} \right) \xrightarrow{R_3 \mapsto R_3 - 2R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array} \right) \xrightarrow{R_3 \mapsto 3R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right) \xrightarrow{\begin{array}{l} R_1 \mapsto R_1 - 3R_3 \\ R_2 \mapsto R_2 + R_3 \end{array}}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & -6 & 3 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right) \xrightarrow{R_1 \mapsto R_1 - 2R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -12 & 5 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array} \right)$$

$$\text{So } A^{-1} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

2) Who cares?

$$\left. \begin{array}{l} x + 2y + 3z = 1 \\ 2x + 5y + 5z = 2 \\ 3x + 8y + 6z = 3 \end{array} \right\} (*)$$

can be written as

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}}_A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

So the solution to (x) is

$$x = 1, y = 0, z = 0.$$