

MA180, MA190, MA185

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Algebra & Calculus

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ALGEBRA

Topics

- Elementary number theory
- Matrix theory
- Eigenvalues & Eigenvectors

Context

Internet communications

Geometry & Internet communications

Breeding rabbits

# Elementary Number Theory (2)

$$5 + 15 = 20$$

In  
School

$$7 + 10 = 5$$

on a  
clock

$$7 + 10 \equiv 5 \pmod{12}$$

Today is Monday. In  
73 days time it will  
be Thursday.

$$73 + 1 \equiv 4 \pmod{7}$$

More examples

$$10 \times 5 \equiv 2 \pmod{12}$$



$$7 + 5 \equiv 3 \pmod{9}$$

3

$$7 \times 8 \equiv 2 \pmod{9}$$

$$2 - 5 \equiv 5 \pmod{8}$$

What is  $\frac{1}{3}$ ?

$\frac{1}{3}$  is that number with the property

$$\left(\frac{1}{3}\right) \times 3 = 1.$$

Alternative notation

$$3^{-1} = \frac{1}{3}$$

we call  $3^{-1}$  the inverse of 3.

## Back to clocks

4

What is  $7^{-1} \bmod 10$ ?

$$7^{-1} \equiv 3 \bmod 10$$

because  $7 \times 3 \equiv 1 \bmod 10$ .

## Application

Any book is identified by an ISBN. On older books this is a string of 10 digits.

0 1 4 1 3 1 1 3 5 5

James and the giant peach.

The final digit is chosen so that

$$(1 \times 0) + (2 \times 1) + (3 \times 4) + (4 \times 1) + (5 \times 3) +$$

$$(6 \times 1) + (7 \times 1) + (8 \times 3)$$

$$+ (9 \times 5) + (10 \times 5)$$

$$\equiv 0 \pmod{11}$$

A second example

$$3 \quad 8 \quad 8 \quad 0 \quad 3 \quad 4 \quad 2 \quad 5 \quad 9 \quad 8$$

$$1.3 + 2.8 + 3.8 + 4.0 + 5.3 + 6.4$$

$$+ 7.2 + 8.5 + 9.9 + 10.8 \equiv 0$$

$$\pmod{11}$$



$$3 + 5 + 2 + 0 + 4 + 6y$$

$$+ 3 + 7 + 4 + 3 \equiv 0$$

$$\text{mod } 11$$

$$9 + 6y \equiv 0 \quad \text{mod } 11$$

$$6y \equiv -9 \quad \text{mod } 11$$

$$6y \equiv 2 \quad \text{mod } 11$$

$$6^{-1} \cdot 6y \equiv 6^{-1} \cdot 2 \quad \text{mod } 11$$

$$y \equiv 2 \cdot 2 \quad \text{mod } 11$$

$$\boxed{y = 4}$$