NUI Galway
OÉ Gaillimh

Semester II Examinations 2013/14

Exam Code(s)	1BMS1, 1BPT1, 1BS1, 1EH1, 1FM1, 1MR1, 1BA1, 1BME1
Exam(s)	First Year Arts and First Year Science
Module(s)	MATHEMATICS
Module Code(s)	MA180, MA186
External Examiner(s)	Dr C. Campbell
Internal Examiner(s)	Dr G. Pfeiffer
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Instructions:	Answer ALL SIX questions.
Duration	Two Hours
No. of Pages	Three Pages
Requirements:	
Release to Library	Yes

1.

(a) A function A is defined for $x \geqslant 2$ by

$$A(x) = \int_2^x \sqrt{e^t}\, dt.$$

i. What is $A'(4)$?
ii. What is $A''(4)$?

(b) Evaluate the following integrals.

$$\text{i. } \int e^x \cos(2x)\, dx \qquad \text{ii. } \int \frac{7x - 24}{x^2 - 6x + 8}\, dx \qquad \text{iii. } \int_2^\infty x e^{-x^2}\, dx$$

2.

(a) Give an example of
i. A subset of $\mathbb{R}$ whose cardinality is 3;
ii. An infinite subset of $\mathbb{R}$ that is bounded;
iii. A countable subset of $\mathbb{R}$ that is bounded above but not below;
iv. A (non-empty) bounded subset of $\mathbb{R}$ that has neither a maximum element nor a minimum element.

(b) State what it means for an infinite set to be *countable*.
Who first proved that the set of real numbers is *not* countable?

(c) Let $S = \left\{ \dfrac{2n+4}{3n} : n \in \mathbb{Z},\ n \geqslant 1 \right\}$.
i. List four elements of S.
ii. Identify, with explanation, the maximum element of S.
iii. Show that S has no minimum element, and determine the infimum of S.

3.

(a) State whether each of the following assertions is *true* or *false*:
i. Every unbounded sequence of real numbers is divergent;
ii. Every increasing sequence of real numbers is unbounded;
iii. If a sequence of real numbers is bounded and convergent, then it is monotonic;
iv. If a sequence of real numbers is bounded and monotonic, then it is convergent.

(b) State what it means for a sequence $(a_n)_{n=1}^\infty$ of real numbers to *converge*. Determine whether the following sequences converge:

$$\text{i. } (\cos n)_{n=1}^\infty \qquad \text{ii. } \left(\frac{1}{n} \sin n\right)_{n=1}^\infty$$

(c) A sequence $(a_n)_{n=1}^\infty$ of real numbers is defined by

$$a_1 = 6,\ a_n = \frac{1}{4}(2a_{n-1} - 3) \text{ for } n \geqslant 2.$$

i. Write down the first four terms of the sequence.
ii. Show that the sequence is bounded below by $-\frac{3}{2}$.
iii. Show that the sequence is monotonically decreasing.
iv. State why it can be deduced that the sequence is convergent, and determine its limit.

4.

(a) Use symbols to write down the logical form of the following argument, then decide whether the argument structure is valid or invalid:

You wear a hat if and only if you wear a tie.
If you wear a hat then you won't wear a jacket.
Therefore you either wear a jacket or you wear a tie, but not both.

(b) (i) Name and define the three properties a relation R on a set S must have to be an equivalence relation. (ii) Show that the relation R defined on the set $S = \{1, 2, 3, \ldots, 16\}$ as the set of pairs

$$R = \{(m, n) \in S \times S : m = 2^k n \text{ for some integer } k\}$$

is an equivalence relation. (iii) Determine the equivalence classes of the elements 1, 2 and 3. (iv) Into how many distinct equivalence classes does this relation R partition the set S?

5.

(a) (i) What is the *order* of a permutation π, what is its *sign*, and how can they be determined from π? (ii) Write the permutation

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 10 & 2 & 11 & 3 & 12 & 9 & 8 & 13 & 5 & 1 & 14 & 6 & 7 & 4 \end{pmatrix}$$

as a product of disjoint cycles. Hence find the order and the sign of π.

(b) (i) Find the quotient and the remainder when the polynomial $x^5 + x + 1$ is divided by the polynomial $x^3 - x^2 - 1$. (ii) Find the irreducible factors of the polynomial $x^3 - 5x^2 + 3x + 1$ in $\mathbb{Z}_{11}[x]$.

6.

(a) State the Principle of Induction. Use it to prove that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all natural numbers n.

(b) Let

$$A = \begin{pmatrix} 6 & -8 & 8 \\ 0 & 6 & 0 \\ 8 & -8 & 6 \end{pmatrix}.$$

(i) Find the characteristic polynomial $p_A(\lambda)$ of the matrix A. (ii) Determine the eigenvalues of A. (iii) Find an eigenvector of A for the eigenvalue 6.